STATE OF ARIZONA
DEPARTMENT OF WATER RESOURCES
ENGINEERING DIVISION

State Standard

for

Supercritical Flow

Under authority of ARS 45-3605(a), the Director of the Arizona Department of Water Resources establishes the following standard for delineation of floodways in riverine environments with supercritical flow in Arizona:

Floodway limits on streams in Arizona which have supercritical flow, for use in fulfilling the requirements of Flood Insurance Studies, and local community and county flood damage prevention ordinances will be determined using the guidelines outlined in State Standard Attachment 3-94 entitled "Floodway Modeling Standards for Supercritical Flow" or by an alternative procedure reviewed and accepted by the Director.

For the purpose of application of these guidelines, supercritical floodway modeling standards will apply to all watercourses identified by the Federal Emergency Management Agency as part of the National Flood Insurance Program, all watercourses which have been identified by a local floodplain administrator as having significant potential flood hazards and all watercourses with drainage areas more than 1/4 square mile or a 100-year estimated flow of more than 500 cubic feet per second. Application of these guidelines will not be necessary if the local community or county has in effect a drainage, grading or stormwater ordinance which, in the opinion of the Department, results in the same or greater level of flood protection as application of these guidelines would ensure.

This requirement is effective December 1, 1994. Copies of this State Standard and State Standard Attachment 3-94 can be obtained by contacting the Department's Engineering Division at (602) 417-2445.

STATE OF ARIZONA
DEPARTMENT OF WATER RESOURCES
ENGINEERING DIVISION

Floodway Modeling Standards

for

Supercritical Flow

500 North 3rd Street
Phoenix, Arizona 85004

(602) 417-2445

STATE STANDARD ATTACHMENT
SSA 3-94

NOVEMBER 1994

Disclaimer of Liability

The methods contained in this publication are intended to be a reasonable way of setting minimum floodplain management requirements where better data or methods do not exist. As in all technical methods, engineering judgement and good common sense must be applied and the methods rejected where they obviously do not offer a reasonable solution.

It must be recognized that while the criteria established herein will generally reduce flood damages to new and existing development, there will continue to be flood damages in Arizona. Where future-condition hydrology (which considers the cumulative effects of development) is not used, future development will probably increase downstream runoff which may result in flooding. Unlikely or unpredictable events such as earthquakes or dam failures may also cause extreme flooding.

The Arizona Department of Water Resources is not responsible for the application of the methods outlined in this publication and accepts no liability for their use. Sound engineering judgement is recommended in all cases.

The Arizona Department of Water Resources reserves the right to modify, update or otherwise revise this document and its methodologies. Questions regarding information or methodologies contained in this document and/or floodplain management should be directed to the local floodplain administrator or the office below:

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Introduction

The National Flood Insurance Program (NFIP) regulations define a floodway as the floodplain area that must be reserved to discharge the base (100-year) flood without increasing the water surface elevation by more than one foot. This NFIP criterion assumes that streams flow at subcritical[1] depth, such that a decrease in floodplain width results in an increase in the flood water surface elevation. However, in high-velocity streams flowing at or below critical depth, a decrease in floodplain width may result in a decrease in water surface elevation. Therefore, the hydraulics of floodway determination for streams with high velocity flow is more complex.

In Arizona, many streams flow near or below critical depth. Steep, bedrock streams may be supercritical at flood stages. Many alluvial streams flow at or near critical depth. Application of subcritical floodway modeling standards to supercritical or near-critical flow may result in unacceptable increases in flow velocity or unsafe encroachment, and may expose future and existing development to excessive flood hazard.

The Arizona Department of Water Resources (ADWR) has established guidelines to be used when modeling floodways for supercritical or near critical flow in Arizona. Accurate floodway delineation for supercritical flow requires special procedures. This document describes the guidelines for modeling types of supercritical floodways for Flood Insurance Studies and floodplain management. In addition, special cases of supercritical flow are described and illustrated in example applications of the guidelines.

When to Apply Guidelines

The guidelines described in this document are to be used for all detailed Flood Insurance Studies and floodplain management applications on streams with supercritical flow in the State of Arizona. These guidelines for supercritical floodway modeling should be applied to streams or stream reaches[2] which meet any of the following criteria:

- A subcritical HEC-2 model of the stream (non-floodway run) defaults to critical depth[3] at three consecutive cross sections, or at 40 percent or more of the cross sections in a reach, or

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[1] For definitions of the terms *"critical," "subcritical,"* and *"supercritical."* see *V.T. Chow, 1959, Open Channel Hydraulics, McGraw Hill Publishing, New York*, or *R.H. French, 1985, Open Channel Hydraulics, 2nd Ed., McGraw Hill Publishing, New York*

[2] A reach may be defined as section of a channel or stream which has similar hydraulic or geomorphic characteristics, such as vegetation, roughness coefficients, area of conveyance, channel geometry, and/or channel slope. Within a reach, cross sections are relatively uniform.

[3] The presence of critical depth should be determined from detailed HEC-2 output, not from the list of error messages at the end of the HEC-2 output printout.

- A subcritical HEC-2 floodway run indicates that the encroached water surface elevation decreases at three consecutive cross sections, or 40 percent or more of the cross sections in a reach, or

- Sound engineering judgement indicates supercritical floodway standards should be applied.

Special Cases of Supercritical Flow

Guidelines for five special cases of supercritical floodway problems are described and illustrated. The five special cases are:

- **Bank Station Designation.** In some cases, the location of the channel bank stations may not be obvious. Because floodways may not encroach within the channel banks of a stream accurate definition of the channel stations is important for floodway modeling.

- **High-Velocity, Near-Critical Flow.** HEC-2 may become computationally unstable at depths near critical depth, and default to critical depth, even where critical or supercritical depth do not occur.

- **Channelized Supercritical Flow.** Where supercritical flow is confined within the designated channel banks, the floodway and floodplain widths are identical.

- **Composite Flow.** Composite flow occurs where both supercritical flow and subcritical flow are present within a single cross section.

- **Braided Flow.** Supercritical flow on braided streams is usually a special case of composite flow, or a case of floodway delineation around islands.

Modeling Guidelines

Appropriate modeling procedures for supercritical floodway modeling may not be intuitively obvious, may require advanced knowledge of hydraulics, and may require minor adjustments for site specific variables. In this document, it is assumed that HEC-2 will be used for floodway modeling. In practice, any hydraulic model which meets local, state, and federal criteria may be used. Modeling guidelines are outlined below.

General Guidelines

These procedures apply to all cases of supercritical floodway modeling outlined in this document. Specific requirements include:

- **Subcritical Profile.** Floodway limits should be determined in the subcritical flow regime when using the HEC-2 program, as required by current FEMA guidelines, regardless of the actual flow regime.

- **Energy Grade Line.** Floodway limits for near-critical or supercritical flow will be determined using the rise in the energy grade line (rather than water surface elevation) caused by encroachment. This corresponds to HEC-2 encroachment method #6.

- **Bank Station Limit.** Floodway limits may not be located inside the channel banks, except in entrenched channels where the entire base flood is contained within the channel banks.

- **Floodway Velocities.** The following comment should be added to the Flood Insurance Study floodway tables when the supercritical flow conditions are present: "Supercritical, or near-critical, flow conditions may exist at the cross sections listed above. The floodway velocities or other velocities shown in this Table should not be used for design purposes, unless an engineering analysis indicates that subcritical flow conditions are present at appropriate cross sections."

- **Floodway Velocity Determination.** Velocities for design and floodplain management purposes should be determined using the supercritical flow option of HEC-2 or an equivalent model. Design velocities should reflect maximum encroachment limits determined using the procedures outlined in this standard.

- **Perched Flow.** These guidelines do not apply to perched flow, except when the perched flow is modeled separately from the main channel floodway. Perched flow originates along well defined channels where overbank flooding becomes separated from the main flow path, and develops hydraulic characteristics unique from the main channel.

- **Roughness Coefficients.** Manning's "N" values should be carefully selected for streams with steep slopes which experience supercritical flow. Manning's "N" values for low gradient streams may not apply. Guidelines for determining "N" values on steep streams are given in Jarrett (1984, 1985).

Channel Bank Designation

In many cases, it is obvious where channel bank stations should be located. Key indicators include the grade break between the bank slope and overbank floodplain, the change in vegetative density between the channel bed and riparian area, or the geomorphic characteristics of the stream. Where channel banks cannot readily be identified from topographic and other data, the Corps of Engineers (1988) definition of channel banks[4] should be used. The Corps defines the channel banks (or the beginning of the overbank area) as the point where depths become less than 3 feet and velocities become less than 3 feet per second. This bank definition may also be used as the starting point for floodway encroachment modeling. It is necessary to perform an initial HEC-2 run to obtain a velocity distribution in order to apply the Corps bank station definition. Subsequent runs will be necessary to refine floodway limits.

For supercritical floodway modeling channel bank stations should be identified using the following:

- **Topographic/Geomorphic Data.** Grade breaks, vegetative and bed sediment characteristics, and channel shape usually help identify bank stations.

- **Hydraulic Data.** Where bank stations cannot be identified from topographic or geomorphic characteristics, the bank station (or the beginning of the overbank) is defined as the point closest to the center of the channel where:

  depth = 3 ft., and
  velocity = 3 ft/s

Example 1: Illustrates Channel Bank Station Designation.

High-Velocity, Near Critical Flow

For streams which flow at or near critical depth, the HEC-2 model may be computationally unstable. Therefore, the modeler should use a optimal number of cross section and data points, as well as verify the accuracy of energy loss coefficients used. HEC-2 critical depth messages may be an indication of unstable modeling, rather than supercritical or critical flow depths. HEC-2 models generally may be regarded as stable

[4] Channel bank definition is intended only for floodway delineation purposes.

if the velocity head is less than 1/3 the flow depth[5]. Where possible, near critical flow models should be calibrated to measured highwater marks.

The following are floodway modeling guidelines and stability tests for high velocity, near critical flow, which supplement the general guidelines outlined above:

- **Velocity Head Criteria.** Compare velocity head and channel depth for channel sections within the stream reach. If the velocity head is less than 1/3 the flow depth (subcritical profile) or greater than 2/3 the flow depth (supercritical profile), the model may be regarded as stable.

- **Additional Cross Section Points.** Compare channel geometry described by ground reference (GR) points relative to upstream and downstream cross sections. Remove or add points to achieve an optimum number of points which accurately describe the section and reach geometry.

- **Energy Loss Coefficients.** Test the sensitivity of the model to variation in energy loss coefficients, such as Manning's roughness coefficients ("N" values). Check model to determine if coefficients selected reflect factors such as bed form roughness, sediment transport, channel slope, and flow depth, as well as bed sediment size, channel shape, and vegetative obstructions.

- **Calibrate.** Obtain high water marks from the channel, where possible, and calibrate computed water surface elevations to the high water mark profile. If an independent estimate of the peak discharge is available, the model can be calibrated using the known discharge as well as the highwater marks.

- **Additional Cross Sections.** Insert new cross sections to determine if flow is actually supercritical or if the model is unstable due to insufficient data.

Example 2: Illustrates Procedures and Output From a Near-Critical Water Surface Profile

Channelized Supercritical Flow

For confined supercritical flow (no overbank flow), floodway (encroachment) modeling should be abandoned. The floodplain limits should be regarded as the floodway boundaries. In some cases, the floodplain limits may be within the channel bank stations defined for the HEC-2 model.

Example 3: Illustrates Two Cases of Channelized Supercritical Flow.

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[5] Corps of Engineers, 1988, "Floodway Determination Using Computer Program HEC-2," Training Document No. 5, Prepared by Vern Bonner, Hydrologic Engineering Center, Davis, California, January, 1988, p. 70.

Composite Flow

For composite flow situations, with supercritical flow in the channel and subcritical or near critical flow in the overbanks, floodway definition may be possible. However, the modeler must ensure that overbank flow modeling is computationally stable using procedures described above. A procedure described in Schoellhamer et. al. (1985) is recommended to determine if composite flow exists. Schoellhamer's procedure involves determining "subdivision Froude numbers" for subdivisions of a cross section. Cross section subdivisions may be the right overbank, left overbank, and main channel, or may be further divided by areas with similar "N" values or by cross section geometry. For cross sections with composite flow, portions of the section will have subdivisions Froude numbers greater than one, and other portions will have subdivision Froude numbers less than one. If composite flow exists and the model is computationally stable, then the floodway may be delineated by assuming the floodway limit is located where overbank depths exceed 3 feet and velocities exceed 3 feet per second, or by applying the guidelines for high-velocity, near critical flow.

The following guidelines are to be used for floodway modeling of composite flow, in addition to the general modeling guidelines outlined above:

- Composite Flow. Use the method of Schoellhamer (1985) to test for the presence of composite flow. It may be necessary to request a trace (J2.10=15) in the HEC-2 input file to use Schoellhamer's procedures.

- Depth/Velocity Limit. Determine if overbank depths and velocities exceed 3 ft. and 3 ft/s, respectively. If these limits are exceeded, and if supercritical flow occurs in the main channel, use the floodplain limits as the floodway limits.

- Additional Cross Sections. Test the model to determine if critical depth message result from insufficient cross sections, or from supercritical flow.

Example 4: Outlines Computations Required to Test for Presence of Composite Flow.

Braided Flow

Application of floodway modeling techniques may not be appropriate for braided streams, and should be considered on a case-by-case basis. Consultation with local floodplain officials and federal agencies is recommended prior to initiating a floodway study for a braided stream. Braided flow, if supercritical flow occurs in flow braids, is essentially a case of composite flow. Therefore, the guidelines for composite flow should be applied. Floodway limits should include all of the flow braids (all of the channel area). Where islands are present between braids, floodway standards for streams with islands should be followed, in addition to supercritical floodway modeling standards. The Corps of Engineers floodway manual, referenced earlier, discusses application of the floodway modeling criteria to braided streams.

Example 5: Illustrates Maximum Encroachment Limits for Streams with Braided Flow.

Works Cited

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Test Applications

Example 1: Channel Bank Designation

- **Problem Statement.** Two channel cross sections are presented in Figures 1 and 2. In Figure 1, channel banks are readily defined by topographic, vegetative, and geomorphic characteristics. In Figure 2, 100-year channel bank stations are less obvious, and the depth/velocity criteria are used. Note that Figure 2 illustrates an example of composite flow.

- **Objective.** Define channel bank stations prior to supercritical floodway modeling.

- **Discussion.** See Figures 1 and 2.

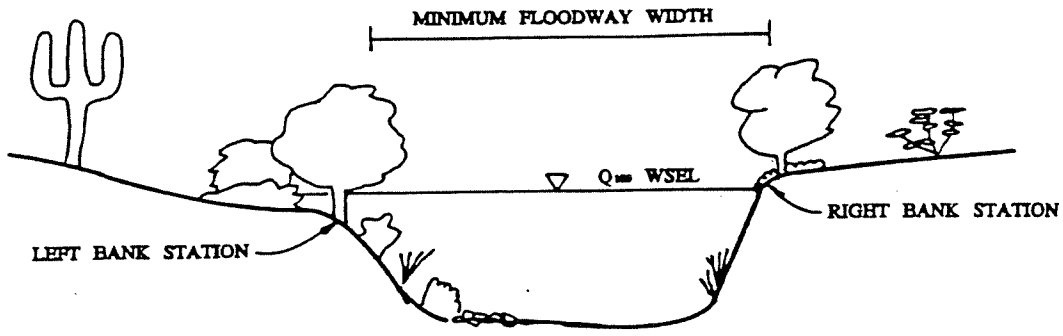


FIGURE 1

CHANNEL BANK STATION DESIGNATION

SIMPLE CHANNEL - DEFINED CHANNEL BANKS USING:

1. SLOPE BREAK
2. VEGETATION

(ILLUSTRATION NOT TO SCALE)

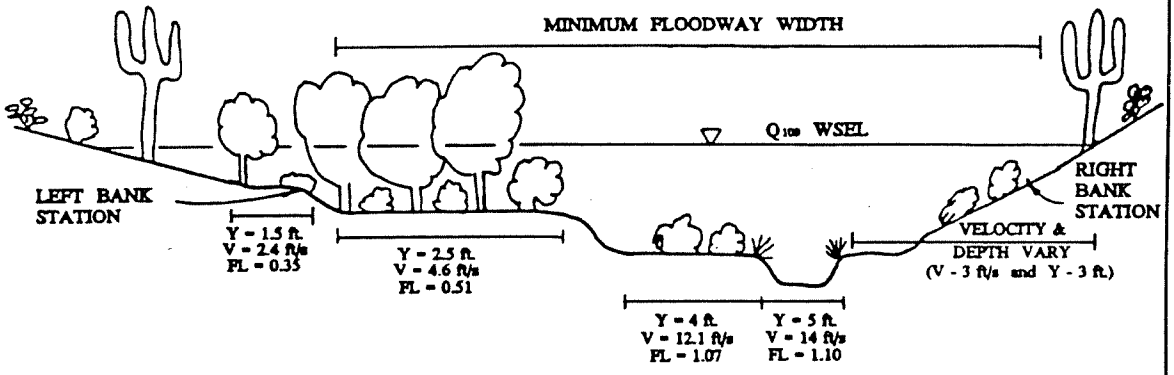


FIGURE 2

CHANNEL BANK STATION DESIGNATION

COMPLEX CHANNEL BANK STATIONS DEFINED AS THE POINTS WHERE FLOW DEPTH BECOMES LESS THAN 3 FT. AND FLOW VELOCITY BECOMES LESS THAN 3 FT/S.

(ILLUSTRATION NOT TO SCALE)

Figure 2 and Figure 2

Example 2: High-velocity, Near Critical Flow

- **Problem Statement.** Cross sections and a plan view profile of a stream is shown in Figures 3 and 4. HEC-2 modeling for a stream indicates critical depth for both subcritical and supercritical profiles, as shown in Figure 5. Tests for stability are outlined. Floodway limits are determined using the energy grade line approach.

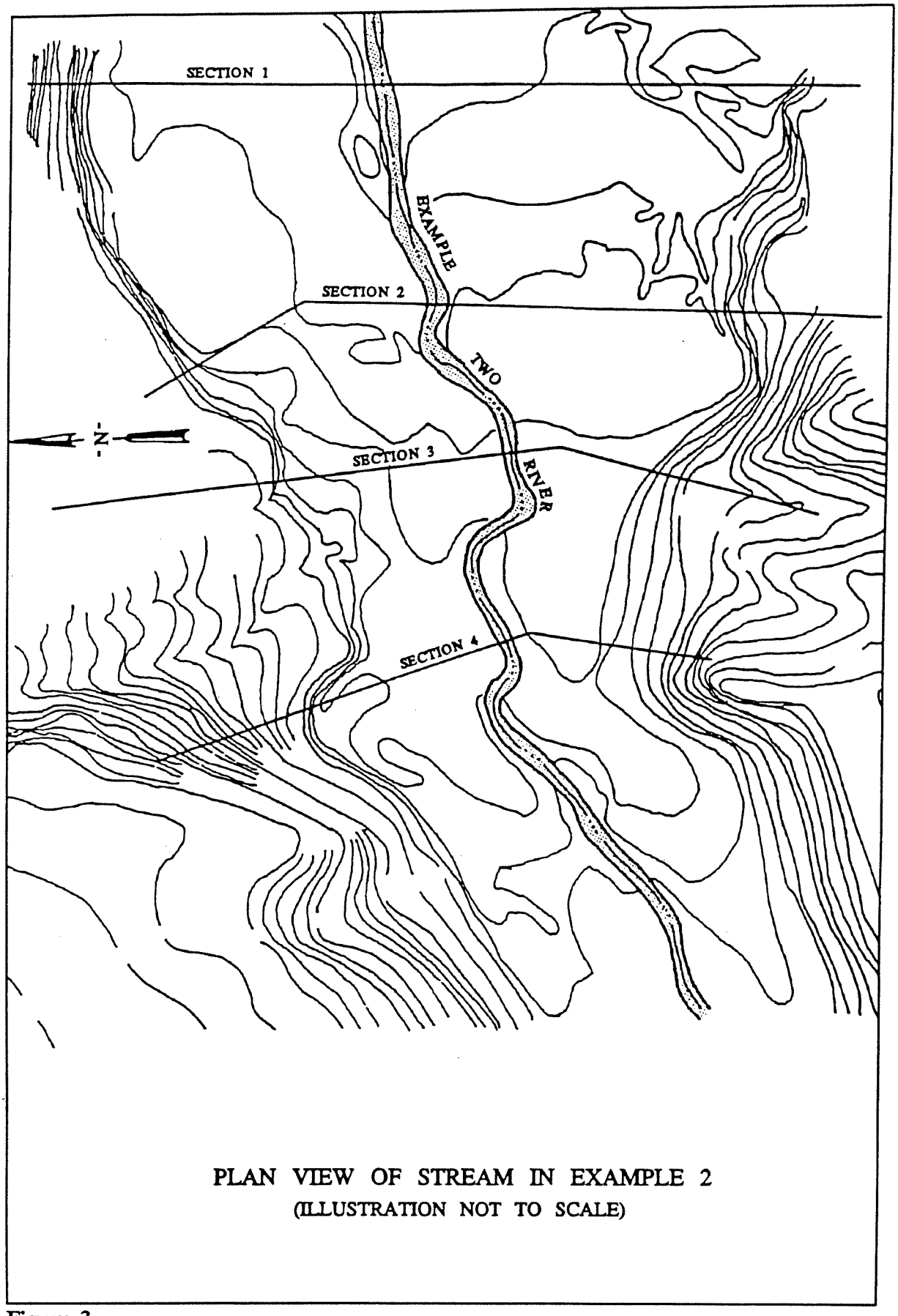
- **Objectives.** (1) Determine if subcritical or supercritical flow occurs, (2) determine if HEC-2 model is computationally stable, and (3) determine floodway limits.

- **Discussion.** The HEC-2 model defaulted to critical depth at three of four cross sections when a subcritical flow regime was assumed (See Table 1). According to the guidelines since more than 40% of the sections were assumed critical, the supercritical floodway modeling guidelines should be used. A supercritical HEC-2 model also assumes critical depth at three of four cross sections (See Table 2). Velocities for both runs average 11.5 feet per second (fps). (However, note the difference in channel velocities computed for the supercritical and critical runs.) Therefore, the profile qualifies as high-velocity, near-critical flow.

  According to the guidelines, additional cross sections should be added, energy coefficients checked, and the model calibrated to insure that the model is computationally stable. A check of the HEC-2 model output indicates that velocity head is less than 1/3 the flow depth for all of the subcritical run. (However, velocity head is not greater than 2/3 the depth for the supercritical run. Therefore, the supercritical run may not be stable.) Additional cross sections were added by interpolation (J1.7=0.1), but did not change computation of critical depth at surveyed cross sections. There is no basis for adjusting energy loss coefficients, or no data for calibration. Therefore, the subcritical HEC-2 model must be assumed to be computationally stable.

  Once the model is checked for stability, the floodway modeling may begin using the subcritical profile HEC-2 model. Encroachment method 6 is used to determine the change in energy grade line, rather than water surface elevation used by method 4, to estimate floodway limits. Encroachment method 6 will not allow encroachment within the channel bank stations. Encroachment stations and floodway data are shown in Table 3. For comparison, floodway data determined using encroachment method 4 are shown in Table 4. Note that use of encroachment method 4 results in a narrower floodway, higher floodway velocities, and decreases in floodway water surface elevation at two of four cross sections. Natural and floodway water surface elevations are shown on the cross section plots in Figures 4a to 4d. HEC-2 input files are shown in Tables 5 through 8.

Note: Floodway velocities for design should be taken from the supercritical run, not the floodway run. Compare Tables 2 and 4.



PLAN VIEW OF STREAM IN EXAMPLE 2
(ILLUSTRATION NOT TO SCALE)

Figure 3

FIGURE 4a
NEAR CRITICAL FLOW
CROSS SECTION 1

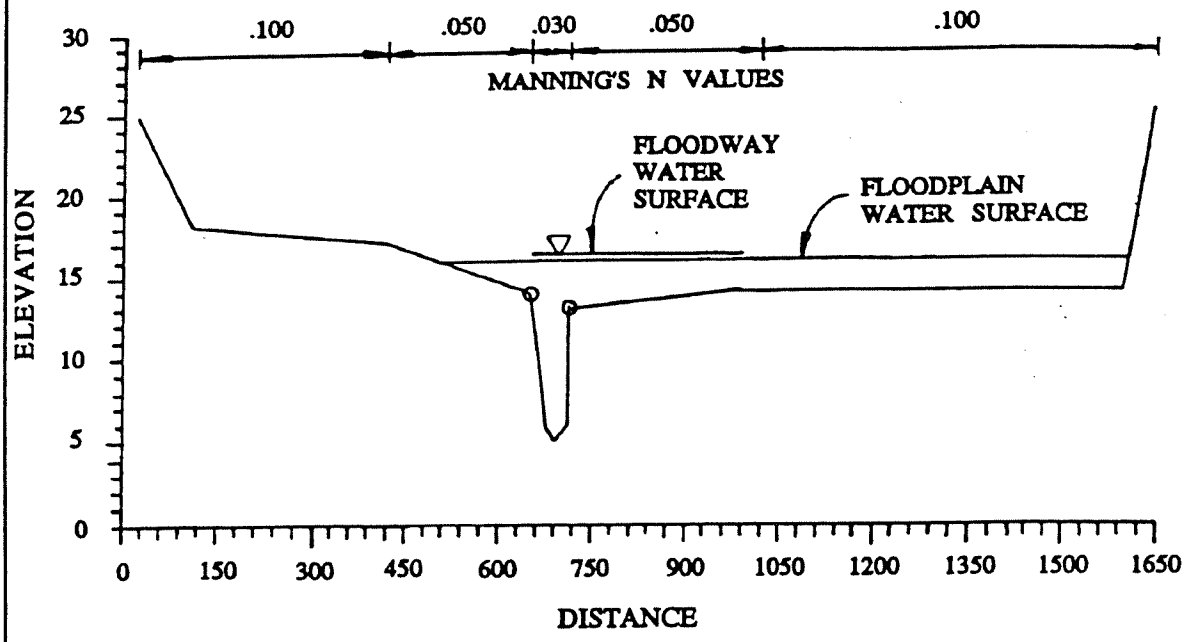


FIGURE 4b
NEAR CRITICAL FLOW
CROSS SECTION 2

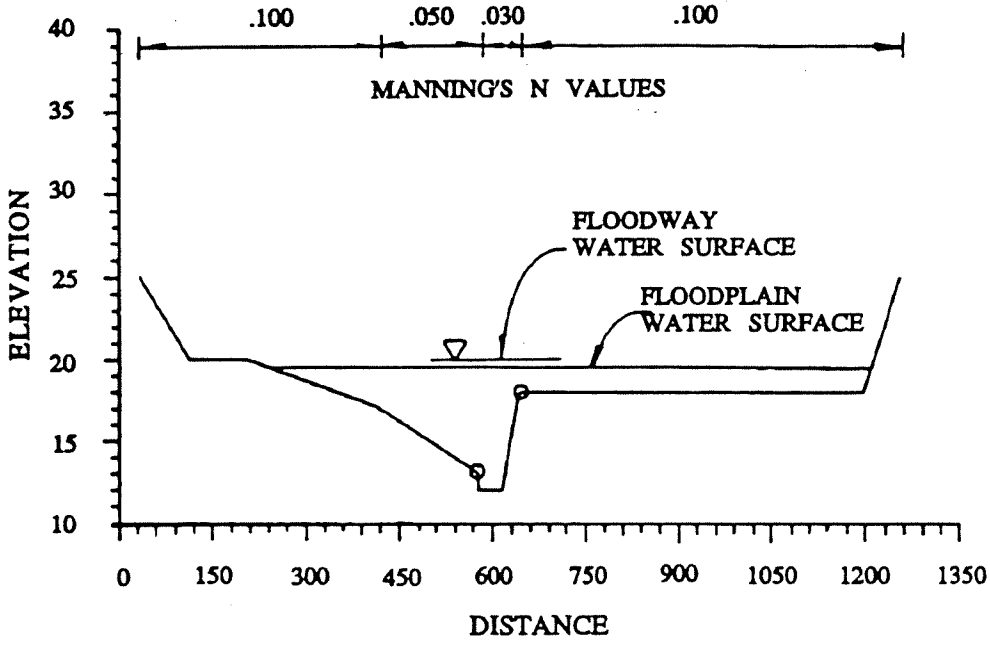


Figure 4

FIGURE 4c
NEAR CRITICAL FLOW
CROSS SECTION 3

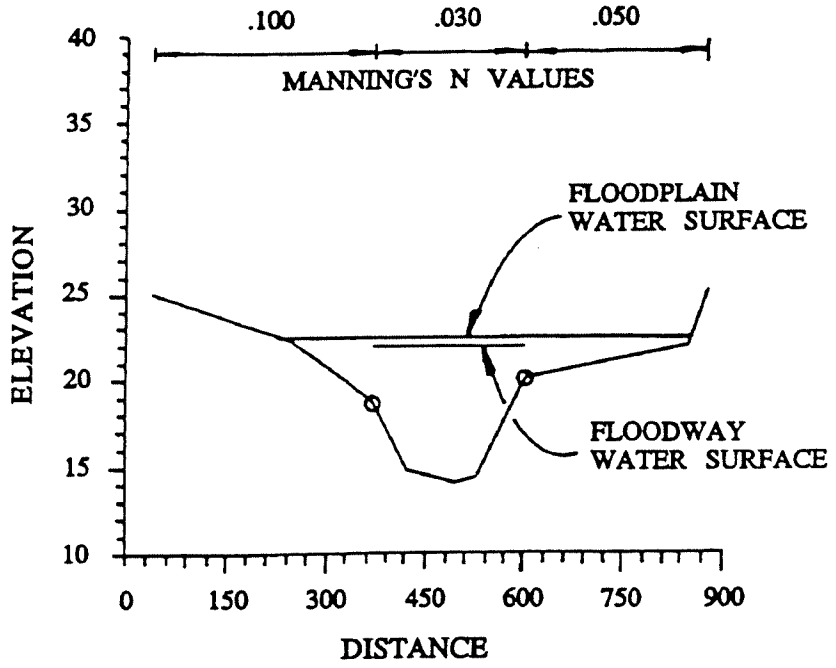


FIGURE 4d
NEAR CRITICAL FLOW
CROSS SECTION 4

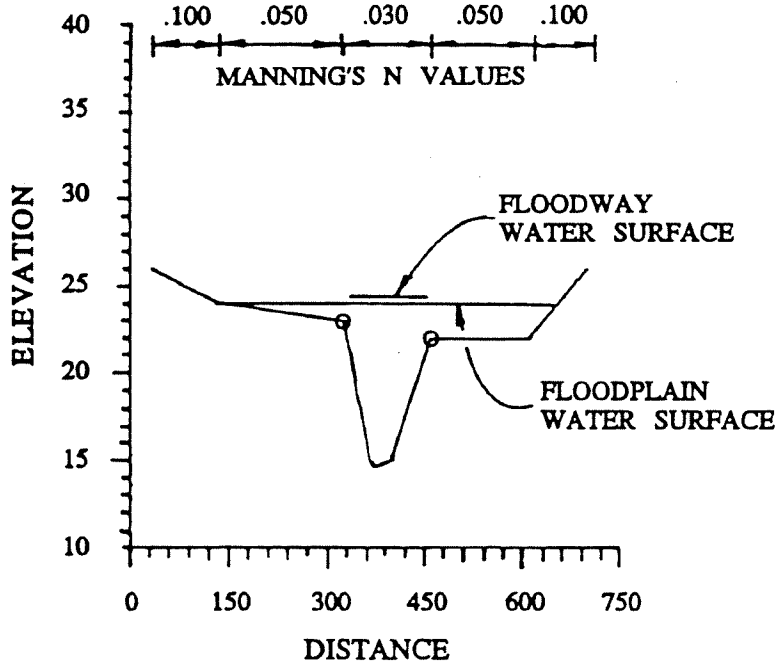
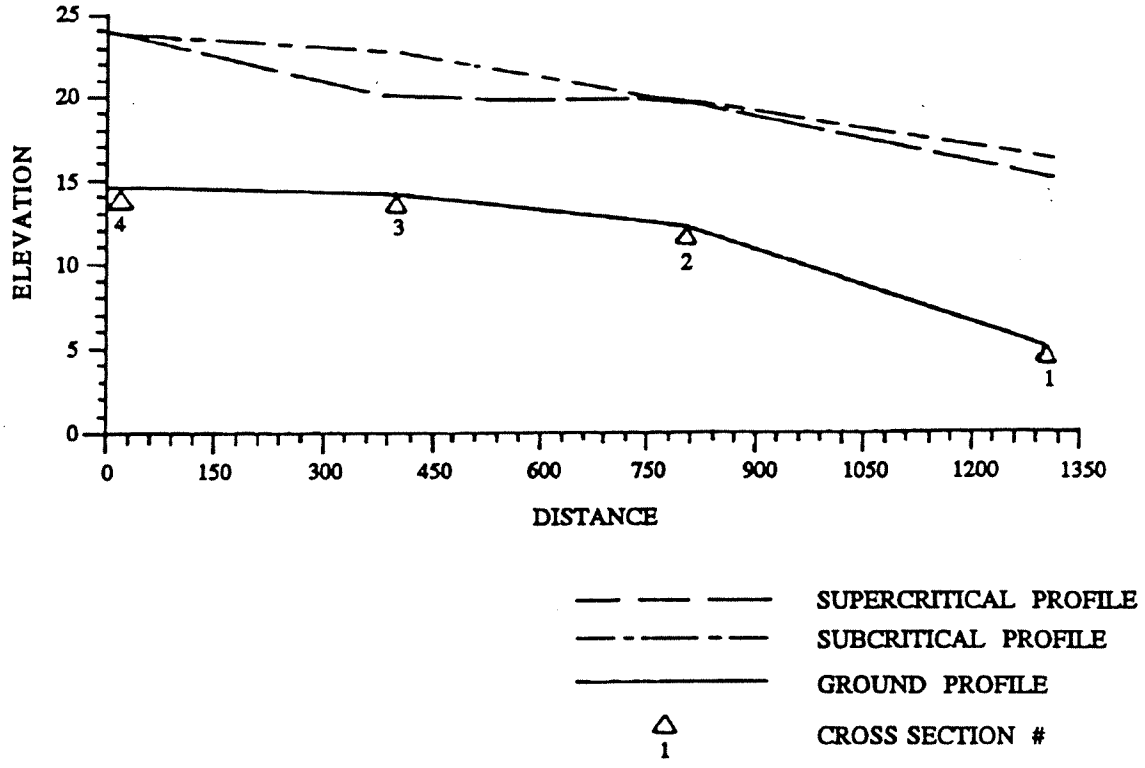


Figure 4



SUBCRITICAL AND SUPERCRITICAL PROFILES
FOR EXAMPLE 2

Figure 5

```
*********************************************
*  HEC-2 WATER SURFACE PROFILES             *
*                                           *
*  Version   4.6.2;  May 1991               *
*                                           *
*  RUN DATE    26APR94    TIME   11:01:39   *
*********************************************
```

```
*****************************************
*  U.S. ARMY CORPS OF ENGINEERS
*  HYDROLOGIC ENGINEERING CENTER
*  609 SECOND STREET, SUITE D
*  DAVIS, CALIFORNIA 95616-4687
*          (916) 756-1104
*****************************************
```

NOTE- ASTERISK (*) AT LEFT OF CROSS-SECTION NUMBER INDICATES MESSAGE IN SUMMARY OF ERRORS LIST

CRITICAL FLOW

SUMMARY PRINTOUT

	SECNO	Q	CWSEL	CRIWS	HV	DEPTH	TOPWID	ALPHA	KRATIO	QCH	VCH	FRCH
*	1.000	10000.00	16.02	16.02	1.09	11.02	1106.85	4.82	.00	5682.49	10.95	.66
*	2.000	10000.00	19.38	19.38	1.18	7.38	961.38	3.29	1.00	4759.89	11.84	.84
*	3.000	10000.00	22.46	.00	.55	8.36	627.57	1.59	2.17	9299.16	6.17	.42
*	4.000	10000.00	23.95	23.95	1.61	9.45	514.23	1.59	1.00	8767.24	10.81	.76

Table 1. Example #2, Subcritical Flow HEC-2 Run Summary Printout.

```
*********************************************
*  HEC-2 WATER SURFACE PROFILES             *
*                                           *
*  Version   4.6.2;  May 1991               *
*                                           *
*  RUN DATE    26APR94     TIME    11:02:05 *
*********************************************
```

```
*************************************
*  U.S. ARMY CORPS OF ENGINEERS
*  HYDROLOGIC ENGINEERING CENTER
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*  DAVIS, CALIFORNIA 95616-4687
*          (916) 756-1104
*************************************
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NOTE- ASTERISK (*) AT LEFT OF CROSS-SECTION NUMBER INDICATES MESSAGE IN SUMMARY OF ERRORS LIST

CRITICAL FLOW

SUMMARY PRINTOUT

	SECNO	Q	CWSEL	CRIWS	HV	DEPTH	TOPWID	ALPHA	KRATIO	QCH	VCH	FRCH
*	4.000	10000.00	23.95	23.95	1.61	9.45	514.36	1.59	.00	8766.73	10.81	.76
*	3.000	10000.00	19.77	19.77	1.96	5.67	262.07	1.04	1.08	9983.10	11.24	1.00
*	2.000	10000.00	19.30	19.30	1.26	7.30	955.27	3.25	1.06	4819.30	12.13	.87
*	1.000	10000.00	14.78	16.06	3.94	9.78	1004.69	4.39	1.07	7929.97	17.84	1.15

Table 2. Example #2, Supercritical Flow HEC-2 Run Summary Printout.

```
*********************************************
*  HEC-2 WATER SURFACE PROFILES             *
*                                           *
*  Version   4.6.2;  May 1991               *
*                                           *
*  RUN DATE    26APR94    TIME   08:59:23   *
*********************************************
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*************************************
*  U.S. ARMY CORPS OF ENGINEERS     *
*  HYDROLOGIC ENGINEERING CENTER    *
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*  DAVIS, CALIFORNIA 95616-4687     *
*        (916) 756-1104             *
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NOTE- ASTERISK (*) AT LEFT OF CROSS-SECTION NUMBER INDICATES MESSAGE IN SUMMARY OF ERRORS LIST

CRITICAL FLOW

SUMMARY PRINTOUT

	SECNO	Q	CWSEL	CRIWS	EG	TOPWID	STENCL	STENCR	DEPTH	HV	QCH	VCH
*	1.000	10000.00	16.02	16.02	17.11	1106.85	.00	.00	11.02	1.09	5682.49	10.95
*	1.000	10000.00	16.24	16.24	18.11	339.49	650.00	989.49	11.24	1.88	6894.38	12.96
*	2.000	10000.00	19.38	19.38	20.56	961.38	.00	.00	7.38	1.18	4759.89	11.84
	2.000	10000.00	19.53	19.24	21.56	260.50	463.87	724.36	7.53	2.03	5715.21	13.88
*	3.000	10000.00	22.46	.00	23.01	627.57	.00	.00	8.36	.55	9299.16	6.17
*	3.000	10000.00	22.08	.00	22.85	230.00	370.00	600.00	7.98	.77	10000.00	7.05
*	4.000	10000.00	23.95	23.95	25.56	514.23	.00	.00	9.45	1.61	8767.24	10.81
*	4.000	10000.00	24.16	.00	26.37	130.00	330.00	460.00	9.66	2.21	10000.00	11.93

Table 3. Example #2, Floodway Encroachment Method 6 HEC-2 Summary Printout.

```
*********************************************
*  HEC-2 WATER SURFACE PROFILES             *
*                                           *
*  Version   4.6.2;  May 1991               *
*                                           *
*  RUN DATE    26APR94    TIME   09:25:42   *
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*****************************************
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*  DAVIS, CALIFORNIA 95616-4687         *
*          (916) 756-1104               *
*****************************************
```

NOTE- ASTERISK (*) AT LEFT OF CROSS-SECTION NUMBER INDICATES MESSAGE IN SUMMARY OF ERRORS LIST

CRITICAL FLOW

SUMMARY PRINTOUT

	SECNO	Q	CWSEL	CRIWS	EG	TOPWID	STENCL	STENCR	DEPTH	HV	QCH	VCH
*	1.000	10000.00	16.02	16.02	17.11	1106.85	.00	.00	11.02	1.09	5682.49	10.95
*	1.000	10000.00	16.34	16.34	18.44	276.16	650.00	926.16	11.34	2.11	7227.82	13.44
*	2.000	10000.00	19.38	19.38	20.56	961.38	.00	.00	7.38	1.18	4759.89	11.84
	2.000	10000.00	19.35	19.32	22.06	167.18	487.44	654.63	7.35	2.70	6204.29	15.50
*	3.000	10000.00	22.46	.00	23.01	627.57	.00	.00	8.36	.55	9299.16	6.17
*	3.000	10000.00	22.59	.00	23.24	230.00	370.00	600.00	8.49	.66	10000.00	6.51
*	4.000	10000.00	23.95	23.95	25.56	514.23	.00	.00	9.45	1.61	8767.24	10.81
*	4.000	10000.00	23.34	23.34	26.24	130.00	330.00	460.00	8.84	2.90	10000.00	13.66

Table 4. Example #2, Floodway Encroachment Method 4 HEC-2 Run Summary Printout.

```
T1 SUPERCRITICAL FLOODWAY STATE STANDARD
T2 EXAMPLE #2 - AKA RED FOX RIVER, HEC2 TRAINING WORKSHOP 3A/3B
T3 NEAR CRITICAL FLOW                SUBCRITICAL RUN
J1              2                    .014              .1              3.8
J2     -1              -1                                                  15
J3     38      43       1       2      10       8       4      57      58      14
J3     26      68
J6      1
QT      1   10000
NC                              .1      .3
NH      5      .1     415     .05     650     .03     710     .05    1020      .1
NH   1635
X1      1      11     650     710
GR     25      20      18     110      17     415      14     650       6     675
GR      5     690       6     710      13     710      14    1020      14    1590
GR     25    1635
NH      4      .1     415     .05     575     .03     640      .1    1250
X1      2      10     575     640     500     500     500
GR     25      30      20     110      20     200      17     415      13     575
GR     12     580      12     615      18     640      18    1195      25    1250
NC     .1     .05     .03
X1      3      10     370     600     400     400     400
GR     25      40      22     260    18.7     370      15     420    14.1     500
GR   14.5     530    17.3     560      20     600      22     850      25     875
NH      5      .1     130     .05     330    .036     460     .05     610      .1
NH    700
X1      4       8     330     460     400     400     400
GR     26      30      24     130      23     330    14.5     370      15     400
GR     22     460      22     610      26     700
EJ
ER
```

Table 5. Example #2, Subcritical Flow HEC-2 Run Data Input File.

```
T1 SUPERCRITICAL FLOODWAY STATE STANDARD
T2 EXAMPLE #2 - AKA RED FOX RIVER, HEC2 TRAINING WORKSHOP 3A/3B
T3 NEAR CRITICAL FLOW                SUPERCRITICAL RUN
J1              2                1    .001              .1              3.8
J2     -1              -1                                                    15
J3     38      43       1       2      10       8       4      57      58      14
J3     26      68
J6      1
QT      1   10000
NC                              .1      .3
NH      5      .1     130     .05     330    .036     460     .05     610      .1
NH    700
X1      4       8     330     460     400     400     400
GR     26      30      24     130      23     330    14.5     370      15     400
GR     22     460      22     610      26     700
NC     .1     .05     .03
X1      3      10     370     600     400     400     400
GR     25      40      22     260    18.7     370      15     420    14.1     500
GR   14.5     530    17.3     560      20     600      22     850      25     875
NH      4      .1     415     .05     575     .03     640      .1    1250
X1      2      10     575     640     500     500     500
GR     25      30      20     110      20     200      17     415      13     575
GR     12     580      12     615      18     640      18    1195      25    1250
NH      5      .1     415     .05     650     .03     710     .05    1020      .1
NH   1635
X1      1      11     650     710
GR     25      20      18     110      17     415      14     650       6     675
GR      5     690       6     710      13     710      14    1020      14    1590
GR     25    1635
EJ
ER
```

Table 6. Example #2, Supercritical Flow HEC-2 Run Data Input File.

```
T1 SUPERCRITICAL FLOODWAY STATE STANDARD
T2 EXAMPLE #2 - AKA RED FOX RIVER, HEC2 TRAINING WORKSHOP 3A/3B
T3 NEAR CRITICAL FLOW        FLOODWAY RUN ENCROACHMENT METHOD 6
J1              2                    .014              .1              3.8
J2      1              -1                                                      15
J3     38      43       1       2       3       4      27      28       8      10
J3     14      26
J6      1
NC                              .1      .3
QT      2   10000   10000
ET                   10.6
NH      5      .1     415     .05     650     .03     710     .05    1020      .1
NH   1635
X1      1      11     650     710
GR     25      20      18     110      17     415      14     650       6     675
GR      5     690       6     710      13     710      14    1020      14    1590
GR     25    1635
NH      4      .1     415     .05     575     .03     640      .1    1250
X1      2      10     575     640     500     500     500
GR     25      30      20     110      20     200      17     415      13     575
GR     12     580      12     615      18     640      18    1195      25    1250
NC     .1     .05     .03
X1      3      10     370     600     400     400     400
GR     25      40      22     260    18.7     370      15     420    14.1     500
GR   14.5     530    17.3     560      20     600      22     850      25     875
NH      5      .1     130     .05     330    .036     460     .05     610      .1
NH    700
X1      4       8     330     460     400     400     400
GR     26      30      24     130      23     330    14.5     370      15     400
GR     22     460      22     610      26     700
EJ
T1 SUPERCRITICAL FLOODWAY STATE STANDARD
T2 EXAMPLE #4 - AKA RED FOX RIVER, HEC2 TRAINING WORKSHOP 3A/3B
T3 COMPOSITE FLOW -      FLOODWAY RUN
J1              3                                      .1            16.02
J2     15              -1                                                      15
ER
```

Table 7. Example #2, Floodway Encroachment Method 6 HEC-2 Run Data Input File.

```
T1 SUPERCRITICAL FLOODWAY STATE STANDARD
T2 EXAMPLE #2 - AKA RED FOX RIVER, HEC2 TRAINING WORKSHOP 3A/3B
T3 NEAR CRITICAL FLOW        FLOODWAY RUN ENCROACHMENT METHOD 4
J1            2                    .014              .1              3.8
J2     1             -1                                                    15
J3    38     43      1      2      3      4     27     28      8      10
J3    14     26
J6     1
NC                           .1     .3
QT     2  10000  10000
ET                10.4
NH     5     .1    415    .05    650    .03    710    .05   1020      .1
NH  1635
X1     1     11    650    710
GR    25     20     18    110     17    415     14    650      6    675
GR     5    690      6    710     13    710     14   1020     14   1590
GR    25   1635
NH     4     .1    415    .05    575    .03    640     .1   1250
X1     2     10    575    640    500    500    500
GR    25     30     20    110     20    200     17    415     13    575
GR    12    580     12    615     18    640     18   1195     25   1250
NC    .1    .05    .03
X1     3     10    370    600    400    400    400
GR    25     40     22    260   18.7    370     15    420   14.1    500
GR  14.5    530   17.3    560     20    600     22    850     25    875
NH     5     .1    130    .05    330   .036    460    .05    610      .1
NH   700
X1     4      8    330    460    400    400    400
GR    26     30     24    130     23    330   14.5    370     15    400
GR    22    460     22    610     26    700
EJ
T1 SUPERCRITICAL FLOODWAY STATE STANDARD
T2 EXAMPLE #4 - AKA RED FOX RIVER, HEC2 TRAINING WORKSHOP 3A/3B
T3 COMPOSITE FLOW -      FLOODWAY RUN
J1            3                                    .1            16.02
J2    15             -1                                                    15
ER
```

Table 8. Example #2, Floodway Encroachment Method 4 HEC-2 Run Data Input File.

Example 3: Channelized Supercritical Flow

- **Problem Statement.** Supercritical flow within two confined channels are illustrated in Figure 6. No floodway analysis is needed, since floodway limits are the floodplain limits.

- **Objective.** Illustrate examples of channelized supercritical flow.

- **Discussion.** Encroachment within the confined channel would be hazardous due to high velocities, the potential to cause hydraulic jumps, and disruption of channel processes. Current federal regulations prevent definition of floodway limits within channel boundaries. Also, only a very limited area within the banks would have depths and velocities less than 3 feet and 3 fps. Supercritical HEC-2 modeling would demonstrate the presence of supercritical flow at most sections in the reach. Floodplain limits would be determined using the subcritical HEC-2 profile. Design velocities should be obtained from the supercritical HEC-2 profile. No floodway modeling would be required.

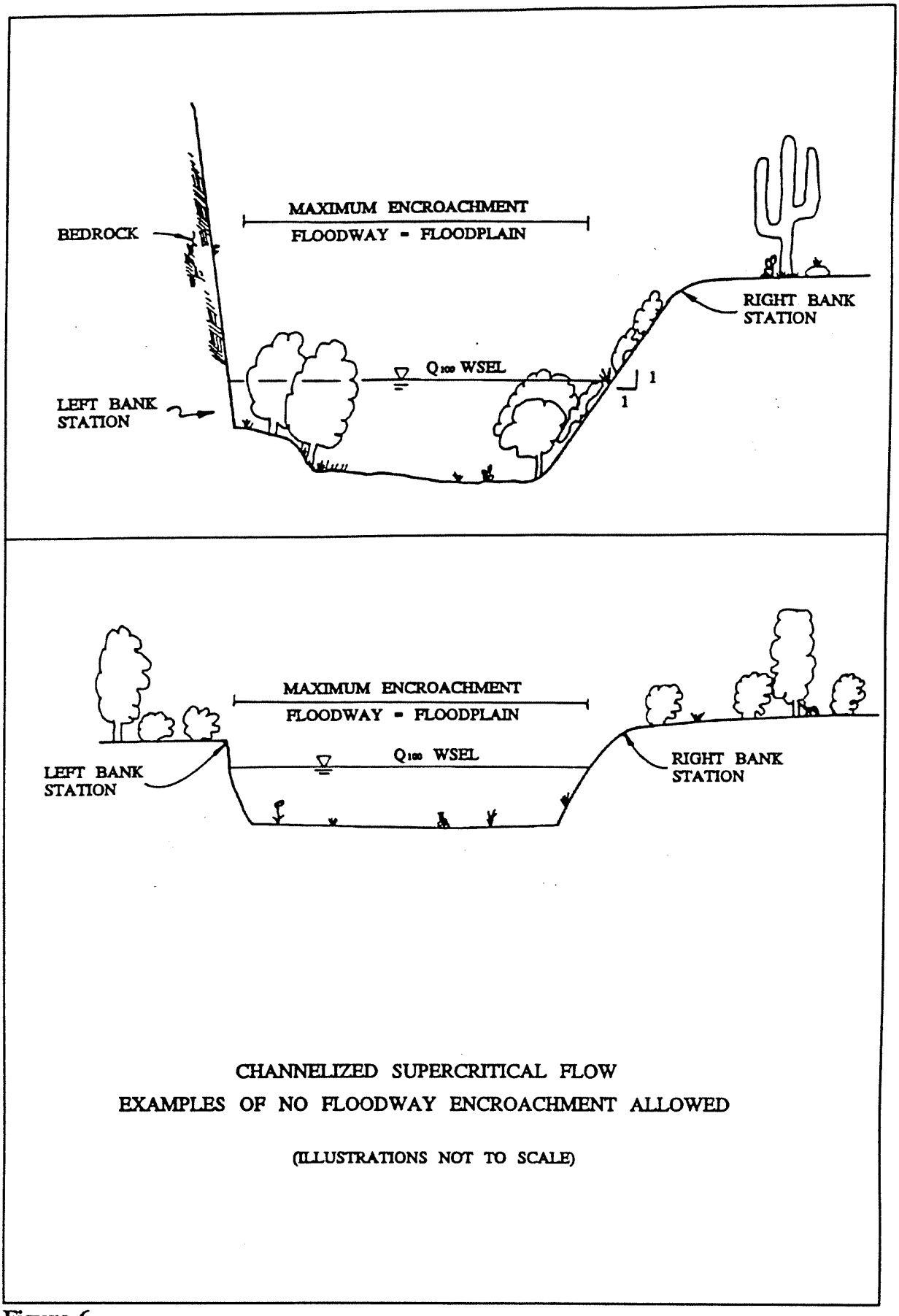
MAXIMUM ENCROACHMENT
FLOODWAY = FLOODPLAIN
BEDROCK
RIGHT BANK STATION
Q100 WSEL
1
1
LEFT BANK STATION
MAXIMUM ENCROACHMENT
FLOODWAY = FLOODPLAIN
Q100 WSEL
LEFT BANK STATION
RIGHT BANK STATION
CHANNELIZED SUPERCRITICAL FLOW
EXAMPLES OF NO FLOODWAY ENCROACHMENT ALLOWED
(ILLUSTRATIONS NOT TO SCALE)

Figure 6

Example 4: Composite Flow

- **Problem Statement.** The stream shown in Example 2 is tested for composite flow. Refer to Figures 3 and 4. Elements of composite flow are illustrated.

- **Objectives.** Demonstrate composite flow tests.

- **Discussion.** The test for composite flow follows the procedure described by Schoellhamer (1986) and uses equations developed in Blalock (1981). Copies of articles by Schoellhamer and Blalock are attached. The example problem is modified from a HEC-2 training problem supplied with the HEC-2 program, and was discussed in Schoellhamer. The procedure involves computation of the subdivision Froude number. The subdivision Froude number describes the ratio of gravitational to inertial forces within segments of a cross section, rather than as an average of the entire cross section. The subdivision Froude number is calculated for each cross section segment to determine if portions are supercritical and portions are subcritical.

  In order to apply the subdivision Froude number procedure, certain hydraulic variables are required. These variables include the total discharge, the energy slope, the topwidth, the left and right end stations of flow, the water surface elevation, cross section conveyance, and total flow area. For the subdivision sections, many of these variables are listed in the detailed output summaries in the HEC-2 output. A trace was requested in the HEC-2 input file (J3.10 = 15) to obtain hydraulic variables for each subdivision of the cross section. Variables requested for output are shown in Table 1 (See Example 2).

  The basic equation for subdivision Froude number is:

$$F_i = \left(\frac{\alpha V_i}{g A_i} \left[\frac{Q_t}{K_t^2} \left(K_i \frac{dK_t}{dy} - K_t \frac{dK_i}{dy} \right) + V_i T_i \right] - \frac{V_i^2}{2g} \frac{da}{dy} \right)^{0.5} \quad ; \textit{where:}$$

F_i = subdivision Froude number, dimensionless
α = velocity coefficient alpha (Coriolis coefficient)
V_i = subdivision velocity, ft/sec
g = gravitational acceleration, ft/sec^2
A_i = subdivision area, ft^2
A_t = total cross section area, ft^2
P_t = total cross section wetted perimeter, ft
P_i = subdivision cross section wetted perimeter, ft
T_i = subdivision topwidth, ft
Q_t = discharge within total cross section, ft^3/sec
K_t = conveyance of total cross section, ft^3/sec
= $(1.49/n_t)A_tR_t^{0.67}$; where:
n_t = Manning's roughness for total section
R_t = hydraulic radius, ft for total section
= A_t/P_t

K_i = subdivision conveyance, ft^3/sec
= $(1.49/n_i)A_iR_i^{0.67}$; where:
n_i = subdivision Manning's roughness
R_i = subdivision hydraulic radius, ft
= A_i/P_i
dK_i/dy = derivative of subdivision conveyance
= $0.33(K_i/A_i)[5T_i - 2R_i\ dp_i/dy]$; where:
dp_i/dy = measured directly, see Blalock (1981)
dK_t/dy =derivative of total conveyance
= $0.33(K_t/A_t)[5T_t - 2R_t\ dp_t/dy]$; where:
dp_t/dy = measured directly, see Blalock (1981)
$d\alpha/dy$ = derivative of the Coriolis coefficient
= $A_t^2s_1/K_t^3 + s_2(2A_tT_t/K_t^3 - A_t^2s_3/K_t^4)$; where:
s_1 = $[(K_i/A_i)^3 (3T_i - 2R_i\ dp_i/dy)]$
s_2 = (K_i^3/A_i^2)
s_3 = $[(K_i/A_i)(5T_i - 2R_i\ dp_i/dy)]$

Subdivision Froude numbers were calculated using the equations shown above for the example cross sections, as shown in Tables 9a-d. Unreal[6] values of the subdivision Froude number indicate subcritical flow. Composite flow was found to exist at each of the sections in the example.

Floodway computations performed.

[6] Unreal, or imaginary numbers, occur when the main term of the basic subdivision Froude number is negative. The square root of a negative number is unreal.

Table 9a
Channel Subdivision
Cross Section 1

GR Station	**491**	**650**	**710**	**1,020**	**1,590**	**1,598**
GR Elevation	**16.02**	**14**	**13**	**14**	**14**	**16.0**
Manning's	n:	0.05	0.03	0.05	0.1	0.1
Discharge	q:	270	5680	2470	1570	10
Flow Area	A:	160.5	519	782.5	1153.8	8.4
Mean Velocity	v:	1.7	10.9	3.2	1.4	0.8
Depth,	y:	1	8.6	2.5	2	1
Topwidth	TW:	159	60	310	570	8
Wetted Perimeter	P:	159	68	310	570	8
Hydraulic Radius	R:	1.0	7.6	2.5	2.0	1.1
Conveyance	k:	4813	99992	43243	27516	129
	dp/dy:	159	0	0	0	8
	S_1:	4205922	1287262432	156956450	23193833	24337
	S_2:	4327965	3711606680	132062820	15649734	30636
	S_3:	14213	57799	85657	67968	349
	dk/dy:	4738	19266	28552	22656	116
Subdivision Froude#	F:	0.45	2.64	0.56	0.21	0.21

Table 9b
Channel Subdivision
Cross Section 2

GR Station	**244**	**415**	**575**	**640**	**1,195**	**1,206**
GR Elevation	**19.38**	**17**	**13**	**18**	**18**	**19.3**
Manning's	n:	0.1	0.05	0.03	0.1	0.1
Discharge	q:	240	3980	4760	1010	10
Flow Area	A:	202.9	700.7	402.2	756.7	7.5
Mean Velocity	v:	1.2	5.7	11.8	1.3	0.8
Depth,	y:	1.2	4.4	6.2	1.4	0.7
Topwidth	TW:	171	160	65	555	11
Wetted Perimeter	P:	171	160	66	555	11
Hydraulic Radius	R:	1.2	4.4	6.1	1.4	0.7
Conveyance	k:	3389	55920	66686	13865	87
	dp/dy:	171	0	0	0	11
	S_1:	499161	243980834	888813847	10241632	27489
	S_2:	945120	356161188	1833235535	4654560	11529
	S_3:	7501	63845	53886	50845	460
	dk/dy:	2500	21282	17962	16948	153
Subdivision Froude#	F:	0.26	0.78	1.81	Unreal #	Unreal #

Table 9c
Channel Subdivision
Cross Section 3

GR Station	**226**	**260**	**370**	**600**	**850**	**854**
GR Elevation	**22.46**	**22**	**18**	**20**	**22**	**22.4**
Manning's	n:	0.1	0.1	0.03	0.05	0.05
Discharge	q:	50	200	9300	500	50
Flow Area	A:	7.8	232.1	1507.3	365	0.9
Mean Velocity	v:	0.2	0.9	6.2	1.4	0.4
Depth,	y:	0.2	2.1	6.6	1.5	0.3
Topwidth	TW:	34	110	230	250	4
Wetted Perimeter	P:	34	110	230	250	4
Hydraulic Radius	R:	0.2	2.1	6.6	1.5	0.2
Conveyance	k:	44	5691	262332	14000	10
	dp/dy:	34	0	0	0	4
	S_1:	15020	4863646	3637511122	42323358	13629
	S_2:	1356	3420764	7946116688	20597368	1204
	S_3:	862	13485	200147	47946	200
	dk/dy:	287	4495	66716	15982	67
Subdivision Froude#	F:	Unreal #	Unreal #	0.54	Unreal #	Unreal #

Table 9d
Channel Subdivision
Cross Section 4

GR Station		**140**	**330**	**460**	**610**	**654**
GR Elevation		**23.95**	**23**	**22**	**22**	**23.9**
Manning's	n:	0.05	0.036	0.05	0.1
Discharge	q:	130	8770	1060	50
Flow Area	A:	90.6	811.2	292.7	42.8
Mean Velocity	v:	1.4	10.8	3.6	1.1
Depth,	y:	0.5	6.2	2	1
Topwidth	TW:	190	130	150	44
Wetted Perimeter	P:	190	130	150	44
Hydraulic Radius	R:	0.5	6.2	2.0	1.0
Conveyance	k:	1647	113868	13624	626
	dp/dy:	190	0	0	44
	S_1:	2337761	1078649320	45374783	144964
	S_2:	544759	2243590585	29513776	133959
	S_3:	13980	91240	34908	1965
	dk/dy:	4660	30413	11636	655
Subdivision Froude#	F:	Unreal #	0.92	Unreal #	0.11

Example 5: Braided Flow

- **Problem Statement.** Figure 7 illustrates a braided flow situation which may or may not have supercritical flow. Maximum floodway limits are defined by the location of flow braids.

- **Objective.** Illustrate maximum floodway encroachment on a braided stream.

- **Discussion.** Since floodway limits cannot be located within designated channel bank stations, the minimum floodway width is the distance between the most distant flow braids. Substantial floodway widths may be defined using these guidelines. For this reason, floodway modeling of braided flow areas should be discussed with local floodplain administrators and review agencies. Where flow braids are separated by significant land areas not inundated by the base flood, modelers should refer to state standards for floodways around islands.

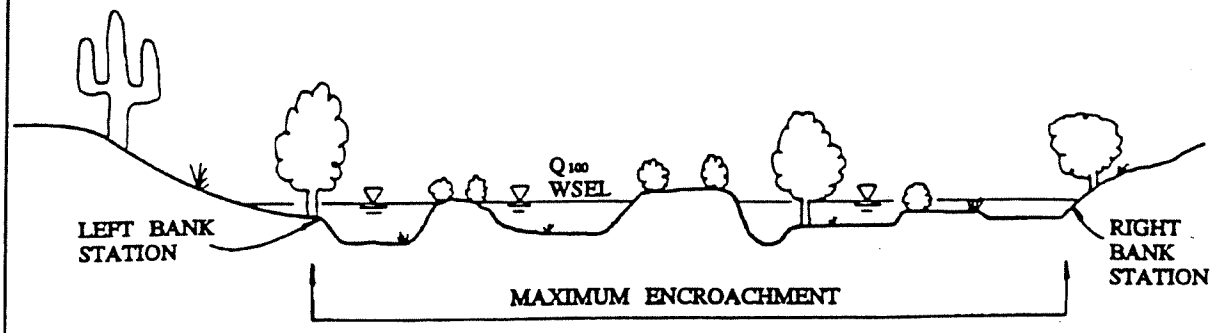

FIGURE 7a
BRAIDED FLOW

PROFILE

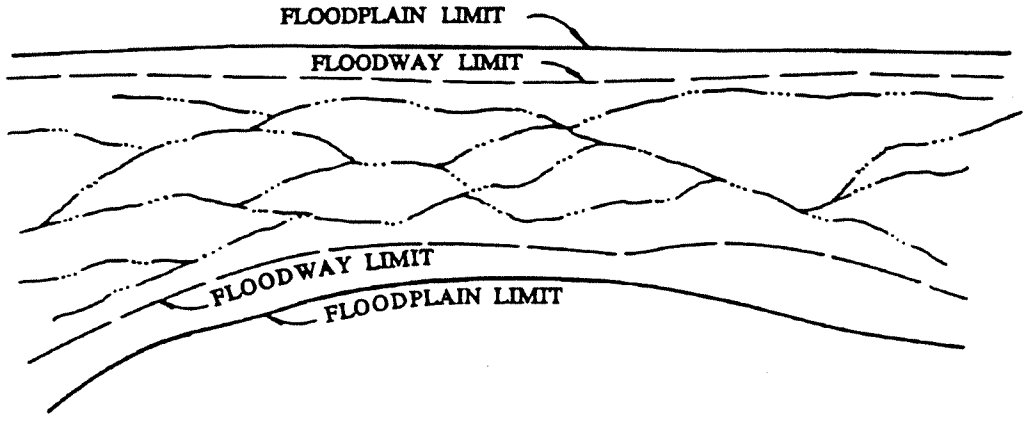

FIGURE 7b
BRAIDED FLOW

PLAN VIEW

MAXIMUM ENCROACHMENT AT MOST EXTREME BRAIDS
(ILLUSTRATIONS NOT TO SCALE)

Figure 7

Appendix

Subdivision Froude Number

By David H. Schoellhamer,[1] A. M. ASCE, John C. Peters,[2] and Bruce E. Larock,[3] Members, ASCE

Introduction

The standard step method calculates one-dimensional steady state water surface profiles by iterating upon the equations for energy conservation and head loss between adjacent cross sections (3). These calculations begin at and proceed away from the controlling boundary cross section. If the flow regime is subcritical the calculations proceed upstream from the downstream boundary, and if the flow regime is supercritical the calculations proceed downstream from the upstream boundary. But this procedure must in some sense be invalid for compound sections in which both flow regimes may occur in different portions of a cross section. Usually when this occurs, the flow in the main channel is in the supercritical regime and the flow in the overbanks is in the subcritical regime (6).

The development and testing of a subdivision Froude number with which the flow regime in each of the three major cross-sectional subdivisions (the two overbanks and the main channel) can be identified is described. This Froude number is compatible with HEC2, a widely used model that employs the standard step method (3,4). The determination of a Froude number for each flow subdivision can enhance the engineer's ability to evaluate the validity of a one-dimensional analysis.

Froude Numbers

The Froude number indicates the flow regime. A value less than one indicates subcritical flow, and a value of greater than one indicates supercritical flow. The simplest definition of the Froude number assumes a uniform velocity distribution so that

$$\mathsf{F} = \frac{V}{\sqrt{gD}} \qquad (1)$$

in which F = Froude number; V = mean velocity; g = gravitational acceleration; and D = hydraulic depth (area divided by top width) (5). A Froude number that considers a nonuniform velocity distribution is

$$\mathsf{F} = V\left(\frac{\alpha}{gD}\right)^{1/2} \qquad (2)$$

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[3]Prof., Civ. Engrg. Dept., Univ. of California, Davis, Calif. 95616.

Note.—Discussion open until December 1, 1985. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on February 27, 1984. This paper is part of the *Journal of Hydraulic Engineering*, Vol. 111, No. 7, July, 1985. ©ASCE, ISSN 0733-9429/85/0007-1099/$01.00. Paper No. 19826.

in which α = Coriolis coefficient. Petryk and Grant (6) developed a Froude number that is the discharge-weighted average of the simple Froude number of Eq. 1 within every subsection. Blalock and Sturm (1) derived a composite Froude number that accounts for the variation of the Coriolis coefficient as a function of the water surface elevation.

Froude number is related to the slope of the specific energy curve. Both Henderson (5) and Blalock and Sturm (2) show for their Froude numbers that

$$\frac{dE}{dy} = 1 - F^2 \quad (3)$$

in which E = the specific energy

$$E = y + \alpha \frac{V^2}{2g} \quad (4)$$

and y = depth. Therefore, when the slope of the specific energy curve is positive, the flow is subcritical, and when the slope is negative, the flow is supercritical.

Subdivision Froude Number

A problem in developing a subdivision Froude number is that the discharge in a subdivision is dependent on the water surface elevation. Therefore the two simple Froude numbers that are defined by Eqs. 1 and 2 are not appropriate for subdivisions of a cross section. Considering subdivision discharge to be a function of the water surface elevation also invalidates the Froude number of Petryk and Grant (6), which Blalock and Sturm (1) showed was inaccurate. Blalock and Sturm's (1) composite Froude number is accurate for an entire cross section, but it is not accurate for subdivisions because it also fails to consider the change of subdivision discharge with water surface elevation.

A subdivision Froude number which allows the discharge to vary with the water surface elevation can be derived from the definition of specific energy. The derivative of specific energy in a subdivision with respect to depth is taken, and both the Coriolis coefficient and the subdivision velocity are assumed to vary with depth. The derivative is substituted into Eq. 3 to arrive at the expression for the subdivision Froude number

$$F = \left\{ \frac{\alpha V_{sd}}{g A_{sd}} \left[\frac{Q}{K^2} \left(K_{sd} \frac{dK}{dy} - K \frac{dK_{sd}}{dy} \right) + V_{sd} T_{sd} \right] - \frac{V_{sd}^2}{2g} \frac{d\alpha}{dy} \right\}^{1/2} \quad (5)$$

in which V_{sd} = subdivision velocity; A_{sd} = subdivision area; Q = cross section discharge; K = cross section conveyance; K_{sd} = subdivision conveyance; and T_{sd} = subdivision top width. The derivatives of subdivision conveyance and Coriolis coefficient are given elsewhere (1,7). The complete derivation of Eq. 5 is given by Schoellhamer (7).

Blalock and Sturm used the same approach to derive their compound Froude number and showed that it was in agreement with experimental results (1). They later stated that use of a celerity that is derived from the method of characteristics produces the identical Froude number (2). Because the compound and subdivision Froude numbers are very sim-

ilar, the method of characteristics would also be expected to show that the subdivision Froude number is correct. In addition, testing shows that the subdivision Froude number is compatible with both the velocity and the specific energy that one finds in a subdivision.

Testing Subdivision Froude Number

The sample trapezoidal cross section of Fig. 1 was initially used to test the subdivision Froude number (7). Five flow rates were tested—100, 1,000, 5,000, 10,000, and 50,000 cfs (1 cfs = 0.028 m^3/s). These flow rates represent extremely low flow, critical depth in the main channel, multiple critical depths, critical depth above the main channel, and extremely high flow, respectively. Each flow rate was tested over a wide range of depths. Two subdivision Froude numbers were calculated, one for the main channel and one for the two identical overbanks. In addition, both the specific energy (Eq. 4) and the derivative of the specific energy were calculated in both subdivisions.

The results of applying the subdivision Froude number to the main channel are very good. For the three largest flow rates, the subdivision Froude number correctly indicates the depth at which the specific energy in the main channel is a minimum, as shown in Table 1. The subdivision Froude number is also compatible with the calculated specific energy for all depths, thus demonstrating the validity of the energy approach used to derive the subdivision Froude number.

The results of applying the subdivision Froude number to the overbank are quite interesting. As shown in Table 2, when the depth in the overbank is very shallow, less than 1.3 ft (0.40 m) for this cross section, the derivative of specific energy with respect to depth is greater than one. This occurs because the velocity head in the overbank increases with depth up to 1.3 ft (0.40 m) and decreases for greater depths. And because the velocity distribution in the overbank is nearly uniform, the velocity behaves like the velocity head. The increase in velocity head over shallow depths in the overbank is intuitively reasonable.

Because the derivative of specific energy is greater than one, Eq. 3 shows that the Froude number squared is equal to a negative number. For this condition Eq. 5 shows that

$$K_{sd}\left(T_{sd} + \frac{A_{sd}}{K}\frac{dK}{dy} + \frac{A_{sd}^3}{2\alpha}\frac{d\alpha}{dy}\right) < A_{sd}\frac{dK_{sd}}{dy} \quad \text{(6)}$$

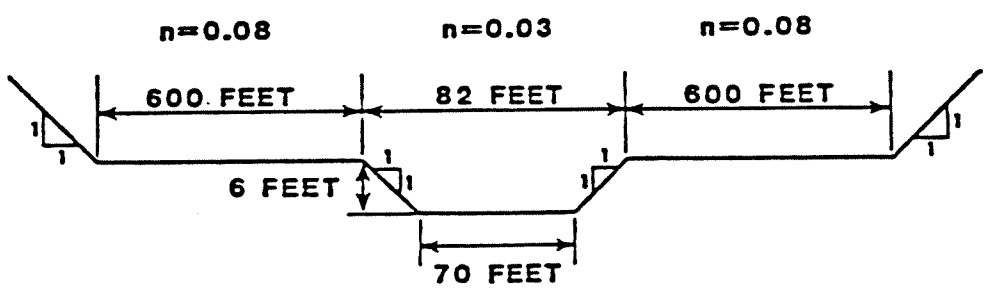

FIG. 1.—Trapezoidal Test Section (1 ft = 0.3 m)

TABLE 1.—Subdivision Froude Number, Main Channel Results[a]

Flow rate (cfs) (1)	Depth (ft) (2)	Subdivision F (3)	E (ft) (4)	dE/dy (5)
5,000	6.5	1.071	7.837	−0.146
	6.6	1.049	7.825	−0.100
	6.7	1.019	7.818	−0.038
	6.8	0.983	7.818	0.034
	6.9	0.944	7.825	0.108
	7.0	0.904	7.839	0.183
10,000	7.9	1.114	9.555	−0.240
	8.0	1.062	9.536	−0.128
	8.1	1.013	9.529	−0.027
	8.2	0.968	9.530	0.064
	8.3	0.924	9.541	0.145
	8.4	0.884	9.559	0.219
50,000	13.1	1.057	16.686	−0.116
	13.2	1.034	16.676	−0.069
	13.3	1.012	16.672	−0.025
	13.4	0.991	16.671	0.018
	13.5	0.971	16.675	0.058
	13.6	0.951	16.683	0.096

[a]1 cfs = 0.028 m^3/s, 1 ft = 0.3 m.

TABLE 2.—Subdivision Froude Number, Overbank Results[a]

Flow (cfs) (1)	Depth (ft) (2)	Velocity (fps) (3)	Subdivision F (4)	E (ft) (5)	dE/dy (6)
5,000	1.0	0.827	[b]	1.011	1.003
	1.1	0.836	[b]	1.111	1.002
	1.2	0.840	[b]	1.211	1.001
	1.3	0.841	0.010	1.311	1.000
	1.4	0.840	0.027	1.411	0.999
	1.5	0.836	0.035	1.511	0.999
10,000	1.0	1.655	[b]	1.043	1.011
	1.1	1.672	[b]	1.143	1.007
	1.2	1.681	[b]	1.244	1.003
	1.3	1.683	0.019	1.344	1.000
	1.4	1.680	0.054	1.444	0.997
	1.5	1.672	0.070	1.543	0.995
50,000	1.0	8.273	[b]	2.063	1.278
	1.1	8.359	[b]	2.185	1.164
	1.2	8.403	[b]	2.296	1.069
	1.3	8.414	0.095	2.399	0.991
	1.4	8.398	0.268	2.495	0.928
	1.5	8.360	0.349	2.585	0.878

[a]1 cfs = 0.028 m^3/s, 1 fps = 0.3 m/s, 1 ft = 0.3 m.
[b]Imaginary number.
Note: The datum for depth and specific energy is the bottom of the overbank.

Eq. 6 shows that the range of depths over which the subdivision Froude number is imaginary is independent of the cross section discharge. This independence has already been implicitly assumed and is confirmed by the results.

When the two sides of Eq. 6 are equal, the subdivision Froude number equals zero and the derivative of specific energy equals one. The depth at which the derivative in the overbank exactly equals one is the depth at which the derivative of the velocity head in Eq. 5 equals zero. This is the depth of maximum overbank velocity head, which for all practical purposes is the depth of maximum overbank velocity, as verified by Table 2.

Thus an imaginary subdivision Froude number indicates that the velocity head is increasing with depth, and therefore the depth in the floodplain is relatively shallow. For this condition it can be concluded that the flow in the overbanks is subcritical because the derivative of specific energy is positive. An imaginary subdivision Froude number may indicate that the overbank flow is too shallow to be modeled properly by the standard step method.

Five test problems containing 193 cross sections were run with a modified version of HEC2 which calculated subdivision Froude numbers. The first test problem was the Red Fox River, which is a problem used by the Hydrologic Engineering Center in training courses on HEC2. Four other test cases were chosen from the test data that is provided to users with each copy of the program (4). These tests (numbers 1, 5, 14, and 15) provided a wide variety of both natural and artificial cross sections. Of the cross sections tested, eleven had a mixed flow regime and 36 had at least one imaginary subdivision Froude number.

Conclusion

A subdivision Froude number has been developed and tested. A knowledge of the magnitude of the subdivision Froude numbers improves the engineer's ability to identify mixed flow regimes and shallow floodplain flow, both of which invalidate the assumptions of the standard step method. A two-dimensional analysis is probably more appropriate in these circumstances.

Acknowledgments

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Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

A = cross section area;
A_{sd} = subdivision area;
D = hydraulic depth (area divided by top width);
E = specific energy;
F = Froude number;
g = acceleration of gravity;
K_{sd} = subdivision conveyance;
K = cross section conveyance (sum of K's);
Q_{sd} = subdivision discharge;
Q = cross section discharge;
T_{sd} = subdivision top width;
V = mean cross section velocity;
V_{sd} = mean subdivision velocity;
y = water depth; and
α = Coriolis coefficient.

Minimum Specific Energy in Compound Open Channel

By Merritt E. Blalock,[1] M. ASCE and Terry W. Sturm,[2] A. M. ASCE

Introduction

Analysis of open channel flow by the application of the energy principle is often clarified and aided by the concept of specific energy, which was introduced by Bakhmeteff (1) in 1912. Specific energy is defined for one-dimensional open-channel flow as the height of the energy grade line above the channel bottom. It leads to a classification of open-channel flow into subcritical and supercritical flow regimes, distinguished by flow depths that are respectively greater or less than the depth at which specific energy is minimum (critical depth). A mathematical consideration of minimum specific energy gives rise to the definition of a Froude number having a value of unity at critical depth. The value of the Froude number is greater than unity for supercritical flow and less than unity for subcritical flow.

The occurrence of critical depth and its associated minimum specific energy is of considerable practical importance to hydraulic engineers. It is one type of channel control which may provide the boundary condition for computation of water-surface profiles in steady, gradually varied flow. Water-surface profile computations are an integral part of water resources investigations involving flood-plain delineations, evaluation of flood control measures, and the design of irrigation and drainage channels.

Petryk and Grant (9) show that the determination of critical depth in channels with overbank or flood-plain flow (compound channels) can be troublesome. Customary definitions of the Froude number generally do not indicate critical depth at the point of minimum specific energy. In addition, there are some compound-channel geometries which produce specific-energy diagrams with two points of minimum specific energy. It is the purpose of this paper to present an analytical formulation of a compound-channel Froude number which correctly identifies the occurrence of points of minimum specific energy for flow in compound open channels. The proposed compound-channel Froude number can

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be used in conjunction with existing computer programs for water-surface profile computation (5,13,16), and is necessarily limited by the same simplifying assumptions that are associated with the conventionally used, one-dimensional equation of steady, gradually varied flow (17). The results of an experimental investigation in a laboratory flume are also presented, demonstrating the existence of two points of minimum specific energy and identifying these points by the proposed compound-channel Froude number.

Froude Number-Flow Regime Discrepancies

For a simple channel of nonrectangular section and uniform cross-sectional velocity distribution, the Froude number F is defined by

$$F = \left(\frac{Q^2 T}{gA^3}\right)^{1/2} \quad (1)$$

in which Q = discharge; T = the top width of the water surface; g = acceleration of gravity; and A = the cross-sectional area of flow. For a compound channel it is customary to include the kinetic energy flux correction coefficient, α, in the definition of specific energy. As a result, α appears as follows in the definition of the Froude number assuming α is constant with depth:

$$F_\alpha = \left(\frac{\alpha Q^2 T}{gA^3}\right)^{1/2} \quad (2)$$

For natural channels with overbank flow, it is often assumed that the major contribution to α is the large difference in mean velocity between main channel and overbank sections. By comparison the nonuniformity of the velocity distribution within each subsection can be neglected.

Two major problems arise in the computation of one-dimensional, steady, gradually varied flow profiles in compound channels as a result of using the Froude numbers F or F_α. First, incorrect solutions are generated when numerical methods are used to solve the gradually varied flow equation written in a form involving the Froude number F_α. Second, incorrect solutions may be accepted when the standard step method is used to compute water-surface profiles near critical depth. These difficulties are the result of neglecting the variation of α with depth in compound-channel flows.

Consider the equation of gradually varied flow in the following form:

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_\alpha^2} \quad (3)$$

in which dy/dx = the rate of change in depth of flow with respect to distance along the channel; S_0 = the bed slope of the channel; and S_e = the slope of the energy grade line. Prasad (10) has proposed a numerical solution procedure for Eq. 3 which can be applied to natural channels. In addition to the assumption that α is constant, the assumptions involved in obtaining Eq. 3 include: no lateral flow, a hydrostatic pressure distribution, a constant bed slope, and a straight, very wide channel, or alternatively, an approximately prismatic channel (17). Because the variation in α with depth and thus with distance along the channel has been neglected, application of Eq. 3 to a gradually varied flow

in a compound channel will lead to incorrect water-surface elevations. The denominator of the term on the right-hand side in Eq. 3 arises from a consideration of the variation of specific energy with depth, a portion of which is due to changes in α with depth in compound-channel flow. Furthermore, the use of F_α can cause the right-hand side of Eq. 3 to become indefinite at a depth that does not correspond to the actual critical depth.

As an alternative to Eq. 3, water-surface profiles are computed in natural channels by the standard step method (6) in which the specific energy is computed explicitly. In this case, F_α does not appear in the equation to be solved, but is used instead to indicate whether the solution is in the supercritical or subcritical flow regime. For compound channels, neither F nor F_α correctly indicates the flow regime. Thus, incorrect solutions of the energy equation can be accepted when the depth is near critical depth.

Compound-Channel Froude Number

Previous Investigations.—Previous investigations of the problems associated with defining the Froude number in compound channels are limited. Numerous laboratory investigations of compound-channel flow have been undertaken (8,11,15), but the focus of these experiments has been the quantification of changes in the boundary shear stress distribution resulting from momentum exchange between the main channel and floodplain. The Federal agencies which maintain and use water-surface profile programs recognize the Froude-number difficulties in compound channels as described in the previous section of this paper, and they examine these difficulties in their user's manuals. The Soil Conservation Service (16), e.g., warns of differences of as much as 2 ft between the critical depth determined by F (Eq. 1) and the critical depth determined by minimum specific energy.

The Corp of Engineers (5) presents an algorithm to solve for the depth corresponding to minimum specific energy when their water-surface profile program attempts to obtain a solution close to critical depth. The depth of minimum specific energy is compared with the profile depth to check the flow regime rather than using the Froude number as a check.

The United States Geological Survey (USGS) (12) proposes the use of an index Froude number based on the Froude number of the subsection carrying the greatest discharge. The index Froude number is thought by the USGS to better reflect the flow regime of the entire cross section, but it is also recognized as having limitations. The USGS does not consider the index Froude number to be a true Froude number, but rather a warning flag that identifies possible flow-regime problems. A later version of the USGS Water Surface Profile Program incorporates a routine to determine the depth of minimum specific energy.

Petryk and Grant (9) have proposed a discharge-weighted Froude number without experimental corroboration in order to eliminate the computational problems associated with the occurrence of two points of minimum specific energy in compound-channel flows. Although their proposed Froude number succeeds in doing this by identifying only one value of critical depth, it is nevertheless somewhat arbitrary and is divorced from the concept of minimum specific energy.

Clearly, the Froude number should be formulated to reflect the specific energy

curve under consideration and should indicate critical depth at the point (or points) of minimum specific energy. Such a Froude number would produce correct numerical solutions of the gradually varied flow equation (Eq. 3) and would eliminate the need for time-consuming routines used to solve for the depth of minimum specific energy in standard step water-surface profile computations.

Derivation and Formulation.—The specific energy, E, for a one-dimensional compound-channel flow is given by

$$E = y + \frac{\alpha Q^2}{2gA^2} \quad (4)$$

in which y = the depth of flow. The kinetic energy flux correction coefficient, α, is defined as

$$\alpha = \frac{\int v^3 dA}{V^3 A} \quad (5)$$

in which v = the velocity through the element of area, dA; and V = the mean cross-sectional velocity (3,6). Alpha is thus a measure of the nonuniformity of the velocity distribution. For computational purposes, flow is conventionally divided into channel and overbank subsections by appropriately located vertical lines which are assumed not to transmit shear stress from one section of flow to another, and which do not contribute to wetted perimeter. Wright and Carstens (15) have suggested that the wetted perimeter of the subsection dividing line be retained for the main channel, and that the shear stress applied by the main-channel flow section on the overbank section be considered. Regardless of the manner in which the main flow-flood-plain interaction is treated, the basic assumption in the computation of α, as previously mentioned, is that the contribution of the nonuniformity of the velocity distribution within each subsection is negligible in comparison to the variation in mean velocity between subsections. If Eq. 5 is applied with this assumption to a compound channel which has been divided into subsections, the kinetic energy flux correction coefficient becomes

$$\alpha = \frac{\sum_i \left(\frac{k_i^3}{a_i^2}\right)}{\frac{K^3}{A^2}} \quad (6)$$

in which k_i = the conveyance of the ith subsection; a_i = the area of the ith subsection; and $K = \Sigma k_i$ = the conveyance of the total cross section (3,6). The subsection conveyance is computed from the Manning equation as follows:

$$k_i = \frac{1.49}{n_i} a_i r_i^{2/3} \quad (7)$$

in which r_i ($= a_i/p_i$) = the subsection hydraulic radius; p_i = the subsection wetted perimeter; and n_i = the subsection n value. In the SI system of units, the constant 1.49 is replaced by unity.

The point (or points) of minimum specific energy is obtained by differentiating Eq. 4 with respect to y and setting the derivative equal to zero. Because both α and area are functions of depth, the differentiation produces (14)

$$\frac{dE}{dy} = 1 - \frac{\alpha Q^2}{gA^3}\frac{dA}{dy} + \frac{Q^2}{2gA^2}\frac{d\alpha}{dy} = 0 \quad (8)$$

Noting that $dA/dy = T$, and that by rearranging terms, the following expression is obtained:

$$\frac{\alpha Q^2 T}{gA^3} - \frac{Q^2}{2gA^2}\frac{d\alpha}{dy} = 1 \quad (9)$$

The left-hand side of Eq. 9 is unity at the point of minimum specific energy; therefore, a compound-channel Froude number F_c can be defined from Eq. 9 as

$$F_c = \left(\frac{\alpha Q^2 T}{gA^3} - \frac{Q^2}{2gA^2}\frac{d\alpha}{dy}\right)^{1/2} \quad (10)$$

At the point of minimum specific energy F_c will have a value of 1.

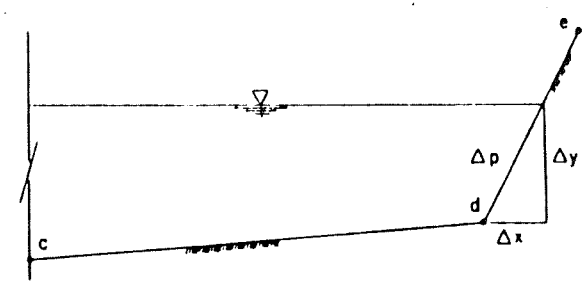

FIG. 1.—Definition Sketch for Evaluation of dp_i/dy

With the exception of $d\alpha/dy$, all of the terms on the right-hand side o Eq. 10 are routinely determined in water-surface profile computations. Evaluatio of $d\alpha/dy$ can be achieved by differentiating Eq. 6 with respect to y. As show in Appendix I, the derivative becomes

$$\frac{d\alpha}{dy} = \frac{A^2\sigma_1}{K^3} + \sigma_2\left(\frac{2AT}{K^3} - \frac{A^2\sigma_3}{K^4}\right) \quad (11$$

in which $$\sigma_1 = \sum_i \left[\left(\frac{k_i}{a_i}\right)^3 \left(3t_i - 2r_i\frac{dp_i}{dy}\right)\right] \quad (12$$

$$\sigma_2 = \sum_i \left(\frac{k_i^3}{a_i^2}\right) \quad (1$$

$$\sigma_3 = \sum_i \left[\left(\frac{k_i}{a_i}\right)\left(5t_i - 2r_i\frac{dp_i}{dy}\right)\right] \quad (1$$

In Eqs. 12-14, t_i = the top width of the ith subsection; and dp_i/dy = the rate of change in wetted perimeter with respect to depth of flow in the ith subsection. Evaluation of dp_i/dy is simplified by the fact that the cross-section geometry of natural channels is defined by ground points connected with straight lines. The definition sketch in Fig. 1 (which is a portion of a right overbank subsection) shows the water surface intersecting the line segment $\overline{de}$. This line segment makes a contribution of Δp to the subsection wetted perimeter. The rate of change in wetted perimeter with respect to depth is a constant along $\overline{de}$, and therefore can be evaluated as

$$\frac{dp_i}{dy} = \frac{\Delta p}{\Delta y} \quad \text{(15)}$$

The terms Δp and Δy are generally determined when computing the geometric properties of a cross section for use in a water-surface profile program. It should be noted that if the water surface is at point e, dp_i/dy should be evaluated for the line segment $\overline{de}$, but if the water surface is at point d, dp_i/dy should be evaluated for the line segment $\overline{cd}$. In situations where the water surface

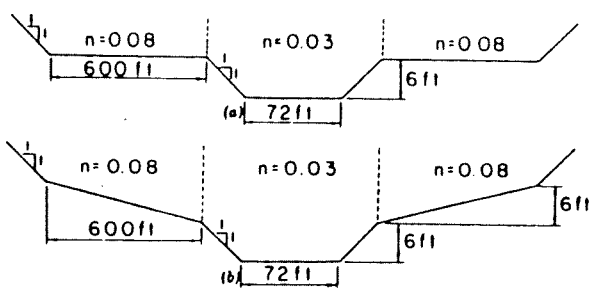

FIG. 2.—Channel Cross Sections for Evaluation of Specific Energy and Froude Numbers: (*a*) Cross Section A; (*b*) Cross Section B (1 ft = 0.3 m)

does not intersect the wetted perimeter of a subsection (e.g., the boundary between the main channel and overbank section above bankfull stage), dp_i/dy is zero. For a subsection where the water surface intersects both a left and right bank (e.g., the main channel below bankfull stage), dp_i/dy is the sum of $\Delta p/\Delta y$ for each of the banks.

The working equation for the compound-channel Froude number can be obtained by substituting Eq. 11 into Eq. 10 and simplifying:

$$F_c = \left[\frac{Q^2}{2gK^3}\left(\frac{\sigma_2\sigma_3}{K} - \sigma_1\right)\right]^{1/2} \quad \text{(16)}$$

If the Manning's n value is considered to vary with depth of flow in any subsection, σ_1 and σ_3 can be written to reflect the variation:

$$\sigma_1 = \sum_i \left[\left(\frac{k_i}{a_i}\right)^3\left(3t_i - 2r_i\frac{dp_i}{dy} - \frac{a_i}{n_i}\frac{dn_i}{dy}\right)\right] \quad \text{(17)}$$

$$\sigma_3 = \sum_i \left[\left(\frac{k_i}{a_i}\right)\left(5t_i - 2r_i\frac{dp_i}{dy} - \frac{a_i}{n_i}\frac{dn_i}{dy}\right)\right] \quad \text{(18)}$$

in which dn_i/dy = the rate of change in n_i with respect to depth of flow.

Evaluation.—The behavior of the compound-channel Froude number, F_c, may be evaluated by examining the specific-energy diagrams of two idealized, symmetric cross sections, each conveying 5,000 cfs (142 m^3/s). Cross section A [Fig. 2(*a*)] is from Petryk and Grant (9). In Fig. 3, the specific-energy curve for this cross section reveals two points of minimum specific energy at depths of flow of approx 6.8 ft (2.07 m) and 5.3 ft (1.62 m). These points are indicated by C1 and C2, respectively, in Fig. 3.

F_c (Eq. 16) for this cross section is plotted in Fig. 4 along with F (Eq. 1) and F_o (Eq. 2). As expected, all three equations produce the same curve below top of bank (simple channel situation), but only Eq. 16 for F_c correctly locates

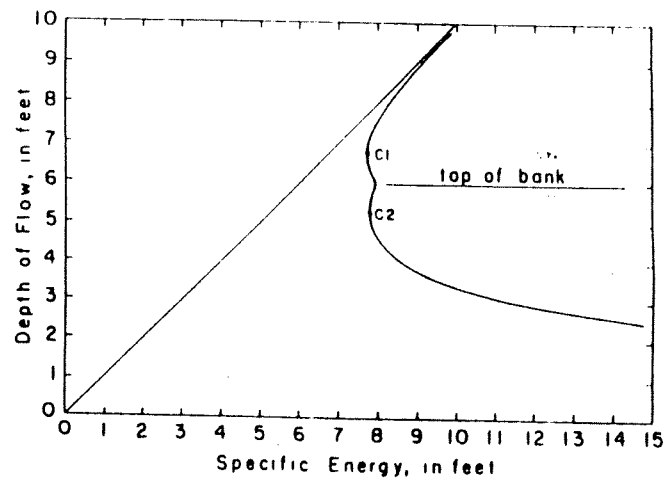

FIG. 3.—Specific Energy for Cross Section A Conveying 5,000 cfs (1 cfs = 0.028 m^3/s; 1 ft = 0.3 m)

C1, the upper depth of minimum specific energy (6.8 ft or 2.07 m), and connects with the lower curve at the top of bank depth.

The shape of the Froude number curve is independent of the discharge, and the fiducial point (F_c = 1) can be shifted left or right by varying the discharge. This means that once F_c is plotted for a particular cross section and discharge, points of minimum specific energy for other discharges may be determined without the necessity of constructing new specific-energy diagrams. In effect, the variable F_c/Q provides a universal horizontal scale for Fig. 4 which depends only on the conveyance and geometric properties of the particular cross section. Thus, for a given depth of flow, the critical discharge, Q_c, can be computed by taking the reciprocal of the corresponding value of F_c/Q, because F_c/Q for the given depth equals $1/Q_c$ for the critical condition.

Cross section B is presented in Fig. 2(*b*) and differs from cross section A only in that the flood plains have a 100:1 slope toward the channel. The

specific-energy diagram of cross section B (Fig. 5) reveals a single point of minimum specific energy below top of bank at the same depth of flow as for cross section A (point C2). The three Froude number curves shown in Fig. 6 for cross section B are again identical below top of bank, but F (Eq. 1) and F_a (Eq. 2) each indicate another point of minimum specific energy above top of bank at depths of flow of 6.5 ft (1.98 m) and 6.8 ft (2.07 m), respectively. The occurrence of these false points of minimum specific energy is a more serious deficiency of Eqs. 1 and 2 than the errors in critical depth shown in Fig. 4.

It is evident from these two examples that the Froude numbers generated by Eqs. 1 and 2 are not acceptable for use in the gradually varied flow equation (Eq. 3). Neither definition of Froude number faithfully reflects the specific-energy diagram in overbank flow situations, and either would produce divergence from a correct profile solution. It is equally evident that Eqs. 1 and 2 are not satisfactory

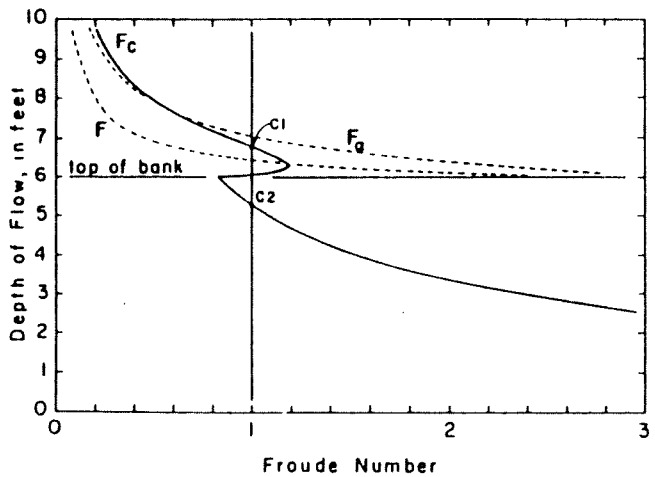

FIG. 4.—Froude Numbers for Cross Section A Conveying 5,000 cfs (1 cfs = 0.028 m^3/s; 1 ft = 0.3 m)

for checking the flow regime in the standard step method. Only F_c (Eq. 17) accurately reflects the specific-energy diagram and indicates the correct flow regime. The experimental investigation into the occurrence of two points of minimum specific energy in the following portion of this paper offers guidance for the interpretation of the flow regime between the two points of minimum specific energy, C1 and C2, in cross section A (Fig. 3).

Experimental Investigation

The experimental investigation consisted of measuring point velocities in a compound-channel cross section which was formed by constructing a single rectangular overbank section in a laboratory flume. Sufficient point velocity measurements were made at eight different depths of flow (at approximately the same discharge for each depth) to compute the discharge, mean velocity,

kinetic energy flux correction coefficient, and specific energy for each depth. Complete details of the experimental procedure are given by the first writer (2).

The experiments were conducted in a tilting steel flume 80 ft (24.38 m) long, 3.5 ft (1.07 m) wide, and 1.5 ft (0.46 m) deep. This flume was also used by Tracy and Lester (14) and details of its construction are given by them. The overbank section was constructed of 3/4-in. exterior plywood and two-by-six fir framing lumber, resulting in the channel dimensions shown in Fig. 7. All wood was coated with sanding sealer and exterior acrylic-latex paint. The overbank section was attached to the flume with silicon adhesive.

Point velocities were measured with a 0.072-in. (1.83-mm) outside diameter pitot-static tube operated in conjunction with a differential pressure transducer. Data collection, reduction, and analysis were accomplished with an HP9825A desktop computer controlling a digital voltmeter which measured the voltage

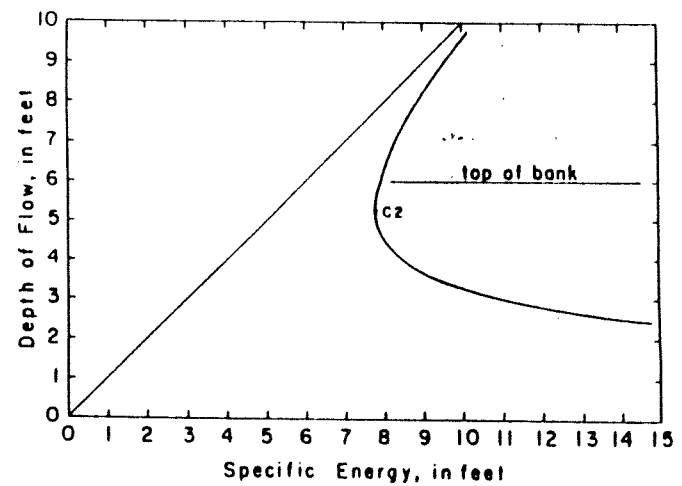

FIG. 5.—Specific Energy for Cross Section B Conveying 5,000 cfs (1 cfs = 0.028 m^3/s; 1 ft = 0.3 m)

output from the pressure transducer and preamplifier. Point velocity measurements were made at a station 65 ft (19.81 m) downstream of the flume entrance. Preliminary measurements were made at a station 60 ft (18.29 m) downstream. Comparison of dimensionless profiles of velocity between the two stations indicated that the flow was fully developed.

The preliminary experiments indicated that a discharge of 1.7 cfs (0.048 m^3/s) would produce a specific-energy curve with two points of minimum specific energy. An estimate of the error in setting the discharge to 1.7 cfs (0.048 m^3/s) included the calibration error of the Venturi meter used to measure the discharge and also included an estimate of the error introduced by observed fluctuations in the Venturi-meter manometer during the course of an experimental run. The estimated error in discharge was of the order of ±3%, which was the same range of error observed between individual discharges determined from the

Venturi meter and the discharges determined by integration of the point velocity measurements.

Establishing a truly uniform flow profile for the experimental runs proved impossible. Any discharge flowing near the depth corresponding to minimum specific energy, as these were, could be expected to be inherently unstable. The instability was exacerbated by the variations in the overbank surface, which were of the order of ±0.01 ft (0.3 cm). Standing waves in the overbank section and a cross-hatched water surface in the channel thwarted efforts to achieve a uniform water-surface profile. As a result, the adopted experimental procedure was to establish a profile as close to uniform as possible such that the desired depth of flow was obtained where the point velocities were to be measured. The maximum observed change in depth for overbank-flow runs was approx 0.05 ft (1.5 cm) between the channel entrance and the measuring station where the flow depth was 0.567 ft (17.3 cm). For larger depths of flow, the water-surface

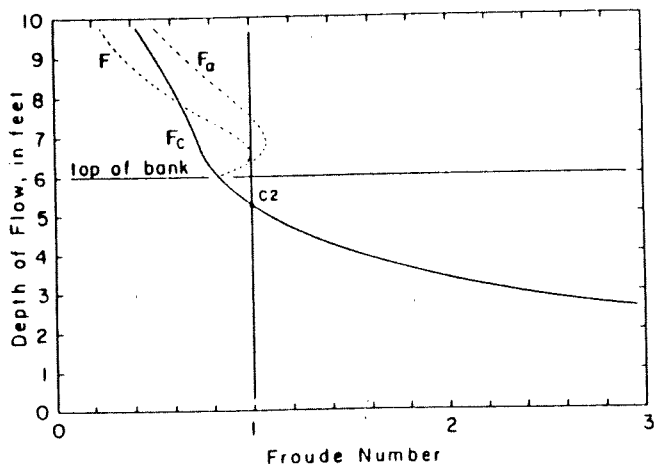

FIG. 6.—Froude Numbers for Cross Section B Conveying 5,000 cfs (1 cfs = 0.028 m^3/s; 1 ft = 0.3 m)

profiles tended to be more stable and more nearly uniform. A profile at a depth of flow of 0.7 ft (21.3 cm) was established to demonstrate that a uniform profile could be obtained in the downstream reach of the flume if the depth of flow was sufficiently greater than the depth corresponding to minimum specific energy.

Results

Table 1 presents the values of area, discharge, kinetic energy flux correction coefficient, and specific energy computed from experimental measurements for each of the eight reported runs. Runs 5 and 6 are not reported in the table because of operational difficulties during each run. It is apparent from the results presented in Table 1 that as the depth increased for those experimental runs with overbank flow, the proportion of the total discharge in the overbank

section increased. It should also be noted that the values of α for the main channel alone are measurably larger than 1.0 because of the narrowness of the main channel section.

Observations of the water surface for the four experimental runs with overbank flow indicated greater instability as the depth of flow decreased. The water-surface instability was manifested by standing waves in the overbank section and a choppy, cross-hatched water surface in the channel section. Beginning at the upper depth of minimum specific energy (run 2) and continuing with decreasing depth, the standing wave fronts in the overbank section were perpendicular to the mean flow direction and then were bent downstream into a cross-hatched pattern in the channel section characteristic of supercritical flow. The surface instability continued to increase for the experimental runs as depth decreased below top of bank. The fact that the water surface was unstable for experimental runs 7 and 8, the first two runs below top of bank in Table 1, suggests that the upper point of minimum specific energy could be considered the limit of subcritical flow for situations in which two points of minimum specific energy occur in water-surface profile computations.

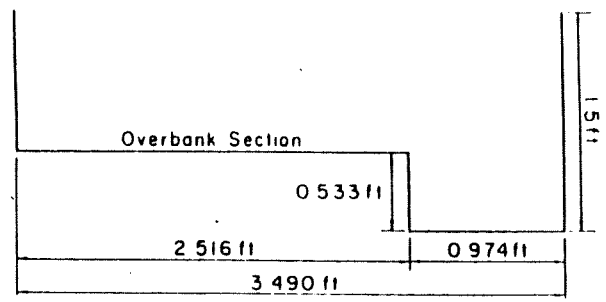

FIG. 7.—Cross Section of Flume and Overbank Section, Looking Downstream (1 ft = 0.3048 m)

The experimental specific-energy data in Table 1 are plotted in Fig. 8(*a*). Although the variation in discharge from run to run causes some scatter in the plot, there is evidence of two points of minimum specific energy. The experimental values of α plotted in Fig. 8(*b*) show little scatter and indicate that α is primarily a function of depth of flow. This observation suggests that a specific-energy diagram for a single value of discharge can be constructed by substituting the average discharge of eight runs (1.692 cfs or 0.048 m^3/s) into Eq. 4 while using the experimental data for all other variables. Fig. 9 presents the resulting average specific-energy diagram. The two points of minimum specific energy are more clearly apparent in this figure.

The concept of computing a Froude number for the flow in a subsection of a compound channel has already been mentioned with regard to the USGS index Froude number (12). The subsection Froude numbers (computed with Eqs. 1 and 2) for the experimental data of this investigation are presented in Table 2. The Froude number of the channel (Col. 3 or 4 of Table 2) is the index Froude number of these experimental runs because the channel is the subsection with the largest discharge. All four depths of flow above top

TABLE 1.—Experimental Data

Run (1)	y, in feet (2)	S_0 (3)	E, in feet (4)	Channel: A, in square feet (5)	Channel: Q, in cubic feet per second (6)	Channel: α (7)	Overbank: A, in square feet (8)	Overbank: Q, in cubic feet per second (9)	Overbank: α (10)	Total: A, in square feet (11)	Total: Q, in cubic feet per second (12)	Total: α (13)
1	0.650	0.001018	0.718	0.633	1.363	1.084	0.294	0.411	1.108	0.927	1.774	1.192
4	0.625	0.001128	0.702	0.609	1.388	1.083	0.231	0.326	1.132	0.840	1.714	1.198
2	0.600	0.001485	0.700	0.584	1.496	1.082	0.169	0.230	1.169	0.753	1.726	1.224
3	0.567	0.002096	0.701	0.552	1.592	1.088	0.083	0.087	1.340	0.635	1.680	1.238
10	0.533	0.001903	0.704	0.519	1.648	1.093				0.519	1.648	1.093
7	0.500	0.002118	0.700	0.487	1.676	1.087				0.487	1.676	1.087
8	0.467	0.003300	0.690	0.455	1.645	1.096				0.455	1.645	1.096
9	0.433	0.004455	0.701	0.422	1.671	1.100				0.422	1.671	1.100

Note: 1 ft = 0.3048 m; 1 cfs = 0.028317 m^3/s.

of bank are subcritical based on the index Froude number, but as shown in Fig. 9, the two lower overbank depths are not subcritical. For this experimental investigation, the index Froude number does not correctly indicate the flow regime of compound-channel flow.

Petryk and Grant (9) apply the concept of a subsection Froude number to obtain their weighted Froude number F_r, which is given by

$$F_r = \frac{\sum_i (q_i F_i)}{Q} \quad (19)$$

in which q_i = the subsection discharge; and F_i = the subsection Froude number computed by Eq. 1. Values of F_r for the experimental data are presented in Col. 7 of Table 2. As in the case of the index Froude number, the weighted Froude number does not correctly indicate the flow regime.

Analysis

The proposed compound-channel Froude number cannot be directly determined from the experimental data. Attempts to use Eq. 10 fail because it is difficult to determine $d\alpha/dy$ from the limited number of experimental data points. Eq. 16 fails because the slope of the energy grade line is not precisely known, which means that the subsection resistance coefficient and thus the conveyance k_i, cannot be determined from the experimental data. If it had been possible to establish a uniform flow condition for each run, the energy gradient would parallel the flume slope, and the conveyance for each subsection could be computed from the experimental data alone. The compound-channel Froude number can only be determined indirectly through an independent prediction of the experimental results.

Working in the same flume as used in the present investigation, Tracy and

Lester (14) experimentally determined a friction-factor relationship for smooth rectangular channels of the form

$$\frac{1}{\sqrt{f}} = 2.03 \log (R\sqrt{f}) - 1.30 \quad (20)$$

in which f = the Darcy-Weisbach friction factor; and R = the Reynolds number. If it is assumed that Eq. 20 is valid when applied independently to each channel subsection, the friction factor, f_i, can be determined for the ith subsection. The mean velocity in the ith subsection, v_i, is then given by

$$v_i = \left(\frac{8gr_iS_e}{f_i}\right)^{1/2} \quad (21)$$

in which r_i = the hydraulic radius of the ith subsection; and S_e = the slope of the energy grade line. Because the values of f_i and v_i obtained from Eqs. 20 and 21 must be such that the subsection discharges sum to the average measured discharge, Q_m, of 1.692 cfs (0.048 m^3/s), the following equation must be satisfied:

$$S_e = \frac{Q_m^2}{\left[\sum_i \left(\frac{8g}{f_i} r_i a_i^2\right)^{1/2}\right]^2} \quad (22)$$

It has been implicitly assumed that S_e is the same for all subsections. Eqs. 20, 21, and 22 can be solved iteratively for the friction factor and velocity in each subsection for a given total discharge and depth. The iterative solution procedure is given in detail by the first writer (2).

The velocities, v_i, were calculated by the procedure just described for the mean measured discharge of 1.692 cfs (0.048 m^3/s). It was assumed that the imaginary vertical boundary between the main channel and overbank section

made no contribution to wetted perimeter. Furthermore, the friction factors determined for each subsection were converted to Manning's n values because the formulation for the compound Froude number, F_c, is in terms of n. The n values so obtained exhibited a slight variation with depth; however, to facilitate the computations, constant n values of 0.009 and 0.010 were adopted for the channel and overbank sections, respectively. From the velocities and n values for each subsection, the specific energy and compound Froude number were computed for a series of depths within the range of measured depths. In the

TABLE 2.—Froude Numbers for Experimental Data

Run (1)	y, in feet (2)	Channel F (Eq. 1) (3)	Channel F_c (Eq. 2) (4)	Overbank F (Eq. 1) (5)	Overbank F_c (Eq. 2) (6)	Weighted F_c (Eq. 19) (7)
1	0.650	0.471	0.490	0.721	0.759	0.529
4	0.625	0.508	0.529	0.821	0.873	0.586
2	0.600	0.583	0.606	0.925	1.001	0.629
3	0.567	0.675	0.704	1.017	1.177	0.692

Note: 1 ft = 0.3048 m.

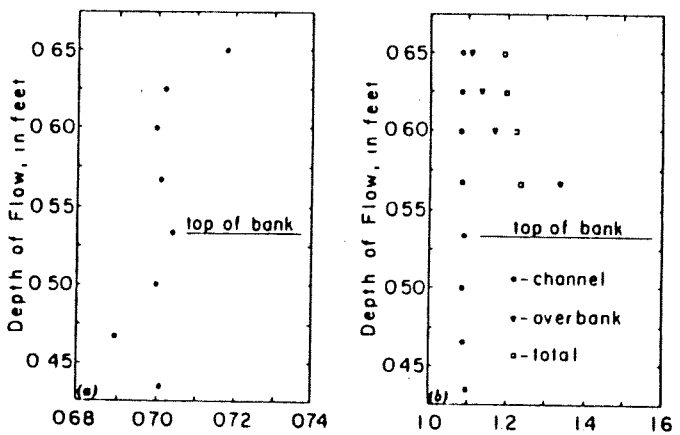

FIG. 8.—Specific Energy and Kinetic Energy Flux Correction Factor from Experimental Data (1 ft = 0.3 m): (a) Specific Energy, in feet; (b) Alpha

computation of the specific energy and F_c, it was assumed that α of each subsection had the value 1.0 rather than the measured value. In this way, the computational procedure remained independent of the measured data and was executed in the same manner as would be expected when determining F_c for a natural river channel in the course of a water-surface profile computation.

The predicted specific-energy diagram is shown in Fig. 10(a), and two depths of minimum specific energy are apparent, although each depth is approximately 2/100 ft smaller than the corresponding depths in Fig. 8(a) or Fig. 9. The

entire specific-energy curve in Fig. 10(a) is skewed slightly downward and to the left when compared with the measured curve in Fig. 8(a) or the average curve in Fig. 9. The predicted compound-channel Froude number curve in Fig. 10(b) exhibits the behavior typical for two points of minimum specific energy,

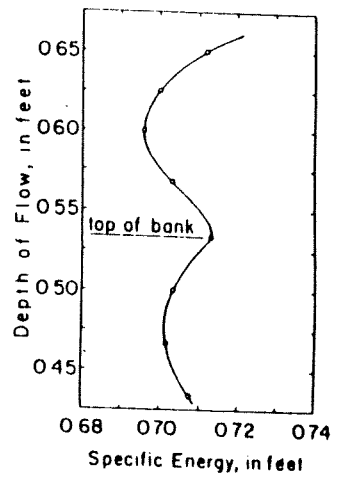

FIG. 9.—Experimental Specific Energy Curve for an Average Discharge of 1.692 cfs (1 cfs = 0.028 m³/s; 1 ft = 0.3 m)

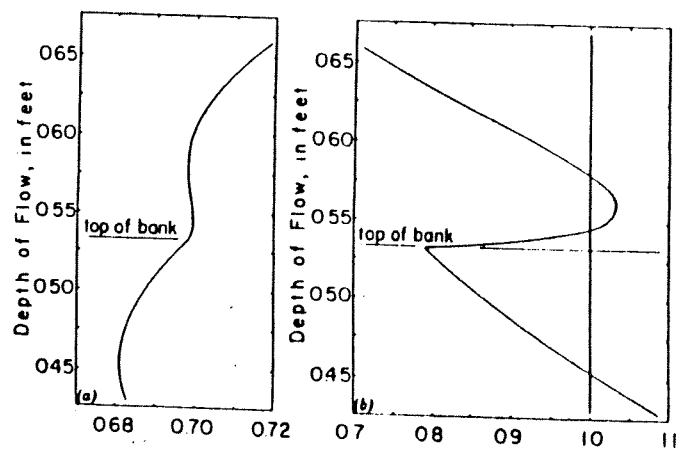

FIG. 10.—(a) Predicted Specific Energy in Experimental Flume for 1.692 cfs; (b) Compound Channel Froude Number for Fig. 10(a) (1 cfs = 0.028 m³/s; 1 ft = 0.3 m)

and is in correspondence with the predicted specific-energy curve as expected.

To investigate the role that neglecting the transfer of linear momentum to the overbank section plays in the skew of the predicted specific-energy curve, the correction suggested by Wright and Carstens (15) was considered. Although

the correction improved the agreement between the measured and computed discharges in the overbank section, especially at the larger depths, the effect on the computed specific-energy curve was minimal because of the relatively small changes in α which resulted from the correction.

The skew in the specific-energy curve is most pronounced below top of bank depth where transfer of linear momentum to the overbank does not occur. The skew in this portion of the curve can be attributed to selecting subsection α values of unity in computing specific energy. It should be noted that the depths of flow in the flume were small compared to depths of flow normally found in field situations. For this reason, the velocity head in the flume makes a large relative contribution to specific energy, and any adjustment to velocity head (such as subsection α) has far more effect on specific energy in the flume than it would in the field.

The same analysis can be applied to subcritical and supercritical flow regimes in field situations where kinetic energy correction coefficients can be as much as 1.4 or more in the main channel (7). For subcritical flow where the velocity head is small, an α-adjustment to velocity head would be insignificant. For supercritical flow, the velocity head can be 50% or more of the depth, and an α-adjustment to velocity head would have a significant effect on specific energy. This reasoning explains the increasing leftward shift in Fig. 10(*a*) as the depth of flow decreases, and the implication is that predicted specific energies and Froude numbers in field channels under subcritical flow conditions would be closer to measured values.

Conclusions

Existing formulations of the Froude number (Eqs. 1 and 2) do not accurately reflect the specific-energy curve for flow in a compound open channel and do not correctly locate points of minimum specific energy. A compound-channel Froude number (Eq. 16) has been derived and has been shown to accurately reflect the specific-energy curve of flow in a compound open channel by correctly locating points of minimum specific energy. When applied to a simple channel with uniform velocity distribution, the compound channel Froude number is identical to Eq. 1, the conventional definition of Froude number.

The compound-channel Froude number is appropriate for use with the gradually varied flow equation (Eq. 3) and provides the proper check of the flow regime when used in conjunction with the standard step method of water-surface profile computation. The proposed Froude number is subject to the same assumptions that apply to the equation of gradually varied flow commonly employed in water-surface profile computations.

For some compound-channel geometries characterized by wide, level flood plains, two points of minimum specific energy can be computed for certain discharges. Laboratory investigation of a one-dimensional flow demonstrates that this phenomenon can in fact occur, and indicates that the upper point of minimum specific energy may be considered the proper limit of subcritical flow.

Acknowledgments

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Appendix I.—Derivation of $d\alpha/dy$

Writing Eq. 6 as

$$\alpha = \frac{A^2}{K^3}\sum_i\left(\frac{k_i^3}{a_i^2}\right) \qquad (23)$$

and differentiating with respect to y produces

$$\frac{d\alpha}{dy} = \frac{A^2}{K^3}\sum_i\left[3\left(\frac{k_i}{a_i}\right)^2\frac{dk_i}{dy} - 2\left(\frac{k_i}{a_i}\right)^3\frac{da_i}{dy}\right] + \sum_i\left(\frac{k_i^3}{a_i^2}\right)\left[\frac{2A}{K^3}\frac{dA}{dy} - \frac{3A^2}{K^4}\frac{dK}{dy}\right] \qquad (24)$$

Noting that $da_i/dy = t_i$, $dA/dy = T$, and $dK/dy = \Sigma_i(dk_i/dy)$, the following is obtained:

$$\frac{d\alpha}{dy} = \frac{A^2}{K^3}\sum_i\left[3\left(\frac{k_i}{a_i}\right)^2\frac{dk_i}{dy} - 2t_i\left(\frac{k_i}{a_i}\right)^3\right] + \sum_i\left(\frac{k_i^3}{a_i^2}\right)\left[\frac{2AT}{K^3} - \frac{3A^2}{K^4}\sum_i\left(\frac{dk_i}{dy}\right)\right] \qquad (25)$$

Evaluate dk_i/dy by writing Eq. 7 as

$$k_i = \left(\frac{1.49}{n_i}\right)\frac{a_i^{5/3}}{p_i^{2/3}} \qquad (26)$$

and differentiate with respect to y to obtain

$$\frac{dk_i}{dy} = \left(\frac{1.49}{n_i}\right)\left[\frac{5}{3}\left(\frac{a_i}{p_i}\right)^{2/3}\frac{da_i}{dy} - \frac{2}{3}\left(\frac{a_i}{p_i}\right)^{5/3}\frac{dp_i}{dy}\right] \qquad (27)$$

Again noting that $da_i/dy = t_i$, and multiplying and dividing by a_i, the following is obtained:

$$\frac{dk_i}{dy} = \frac{1}{3}\left(\frac{k_i}{a_i}\right)\left[5t_i - 2r_i\frac{dp_i}{dy}\right] \qquad (28)$$

Substituting Eq. 28 into Eq. 25 and simplifying, results in Eq. 11.

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Appendix III.—Notation

The following symbols are used in this paper:

A = total cross-section area;
a_i = subsection area;
E = specific energy;
F = Froude number;
F_c = compound-channel Froude number;
F_i = subsection Froude number;
F_r = weighted Froude number;
F_α = Froude number with kinetic energy flux correction;
f = Darcy-Weisbach friction factor;
f_i = subsection friction factors;

g = acceleration of gravity;
K = total cross-section conveyance;
k_i = subsection conveyance;
n = Manning's n value;
n_i = subsection n value;
p_i = subsection wetted perimeter;
Q = total cross-section discharge;
Q_m = average measured discharge;
q_i = subsection discharge;
R = Reynolds number;
r_i = subsection hydraulic radius;
S_e = slope of energy grade line;
S_0 = bed slope of channel or flume;
T = total cross-section top width;
t_i = subsection top width;
V = total cross-section mean velocity;
v = mean velocity associated with incremental area, dA;
v_i = subsection mean velocity;
x = distance along channel;
y = depth of flow;
α = kinetic energy flux correction coefficient;
Δp = increment of wetted perimeter;
Δy = increment of depth; and
$\sigma_1, \sigma_2, \sigma_3$ = subsection parameters of compound-channel Froude number.